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ANALYSIS OF WATER INGESTION EFFECTS IN AXIAL FLOW COMPRESSORS.(U)

JUN 78 S N MURTHY, B A REESE, G T ARCANGELI

F33615-74-C-2014

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**ANALYSIS OF WATER INGESTION EFFECTS  
IN AXIAL FLOW COMPRESSORS**

S.N.B. MURTHY  
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JUNE 1978



TECHNICAL REPORT AFAPL-TR-78-35  
Final Report September 1976 – November 1977

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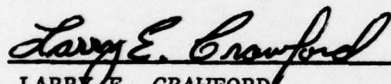
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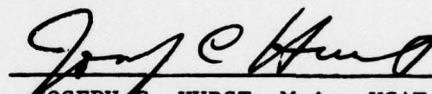
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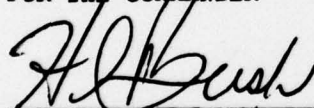
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFAPL TR-78-35	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYSIS OF WATER INGESTION EFFECTS IN AXIAL FLOW COMPRESSORS	5. TYPE OF REPORT & PERIOD COVERED Final 1 Sep 76 - Nov 77	6. PERFORMING ORG. REPORT NUMBER M-WPAFB-T-76-11
7. AUTHOR(s) S. N. B. Murthy, B. A. Reese, G. T. Arcangeli, and T. Tsuchiya	8. CONTRACT OR GRANT NUMBER(s) F33615-74-C-2014	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Purdue University School of Aeronautics and Astronautics West Lafayette, Indiana 47907	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 3048-05-80	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Aero Propulsion Laboratory (TBC) Wright-Patterson AFB, OH 45433	12. REPORT DATE June 78	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 93	15. SECURITY CLASS. (of this report) Unclassified
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Water ingestion; turbomachinery; jet engines; two-phase flow		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In the study of the general problem of predicting performance and instability of installed jet engines during water ingestion (deliberate or accidental), the establishment of the behavior of the compressor in an engine is an important task. In continuation of an earlier study, this report includes (a) analysis of selected compressors with water ingestion and evaporation, (b) effect of bleed and/or injection, and (c) further development of a two-phase model with three droplet sizes for calculating compressor performance. An important result of investigations on selected compressors is the crucial influence of the		

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location of water evaporation in the compressor and the blade loading. A second significant outcome of the investigations is that in order to simulate the engine with two-phase flow component responses are required in great detail.

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## SECTION I

### INTRODUCTION

The problem of water ingestion into engines arises both when water is injected deliberately to increase the thrust output and when water enters an engine accidentally, for example during rain or operation from a rough runway with puddles of water on it. A complete analysis of the problem of water ingestion requires consideration of an installed engine including engine location with respect to the airframe and also the wheels. Thus, one has to establish changes in the performance and stability of the following.

- a. inlet
- b. fan and fan duct
- c. compressor
- d. diffuser
- e. combustor and
- f. afterburner.

In addition, one has to take the effects of water ingestion into account in engine-airframe integration. Among the components, while each of those referred to above is of importance, it is clear that the compressor performance and stability are crucial to the establishment of overall performance. The investigation reported on here is an extension of an earlier study (Ref. 1) on axial flow compressor operation with air-water mixture flow.



### 1.1. Outline of the Report

The three principal topics discussed in the report are (1) analysis of selected compressors with water ingestion and evaporation, (2) engine instability simulation with water ingestion, (3) further development of the axial flow compressor performance calculation program for two phase flow. The later includes a discussion of the method of incorporating variable composition and properties of the working medium into the performance calculation procedure. The three topics are discussed in the following three sections of the Report.

The principal objective of the investigation was the establishment of the major areas in which further research is required in axial flow compressors when operating with air-water mixture flow. The preliminary studies reported on here are intended to identify such outstanding problems and a short discussion on them is provided in the last chapter entitled "Discussion", along with conclusions from investigations on two selected compressors.

## SECTION II

### ANALYSIS OF SELECTED COMPRESSORS

When water is ingested into a compressor, two general questions can be posed as follows.

(i) Suppose the water evaporates at a certain station in the compressor. What is the effect on the remaining stages of the compressor?

(ii) Suppose again that the water evaporates at a certain station in the compressor. If we desire to bring the performance of the stage following that station to the values that existed when operating with air alone, all other conditions being equal, is it possible to achieve this with bleeding and/or injection?

Investigations on these questions follow.

#### 2.1 Influence of Blade Loading

When water evaporates at a certain station in the compressor, it leads to the following, (i) reduction in the temperature of air; (ii) change in the composition and properties of the flowing medium; (iii) change in the corrected speed and mass flow of the compressor (iv) change in the operating points of the stages following the position where the water evaporates.

Consider for example a 6-stage compressor. Suppose we assume that the initial conditions of the air-water mixture at entry to the compressor are such that under a given set of conditions, the water undergoes evaporation between the third and the fourth stages. The performance of the compressor becomes affected in the first three stages due to the

presence of water ingested with air and because of the upstream effects induced by the changes in the successive stages. It is clear that the performance of stages 4, 5, and 6 will become affected because of the reduction in temperature and addition of steam between the third and fourth stages. The performance change will be in regard to the corrected speed and mass flow and pressure ratio, the three quantities that make up an operating point. Thus the overall performance of the compressor will undergo a change.

In presenting the modified performance of the compressor for a given set of operating conditions, it is necessary to adopt a uniform procedure. The one adopted here is the standard procedure of presenting the compressor performance map in terms of the mass flow parameter at entry to the compressor.

#### 2.1.1. Problem formulation

The following assumptions have been introduced.

(1) Water droplets, which occupy a negligible volume in the flowing medium because of the low concentration do not affect the compressor performance. In other words, a compressor operating with air-water mixture operates just as though it was operating with air alone.

(2) When the heat content of air and water are right, water undergoes evaporation instantaneously. Consequently, there is an instantaneous change in the temperature (reduction) and composition (with addition of steam) of the flowing medium. The corrected mass flow and speed become altered and therefore the output of the stages following the evaporation station will undergo change.

(3) The mass of water evaporated is assigned a certain value in each calculation.

(4) In calculating the performance of the compressor after evaporation of water, the performance of various stages of the compressor is based on the original air flow data (Refs. 2 and 3). In other words, in estimating the performance of a stage with air-steam mixture, we utilize the characteristic of the stage as determined with air as the basis while accounting for changes in molecular weight and ratio of specific heats.

Utilizing the foregoing assumptions, the performance of the compressor with evaporation of water is calculated based on the so-called one-dimensional analysis. In that analysis one assigns flow conditions to a certain section along the height of a blade and utilizes performance characteristics corresponding to that section of the compressor to obtain outlet conditions from the stage at that section.

It should be pointed out here that the following investigation pertains entirely to the effects of evaporation of water at a chosen stage in the compressor, leading to a reduction in temperature of the fluid locally, the exchange of mass between the liquid and the vapor phase and the consequent change in the molecular weight and ratio specific heats of the fluid in the compressor. Thus, the effects of the presence of water in some of the cases in certain stages of the compressor and the induced effects of changes in certain stages on other upstream stages have not been investigated here. Insofar as the presence of water is concerned, in view of the small volume of water ingested, in this preliminary analysis, assumption (i) is introduced



so that one has a reasonable basis for carrying out the calculation of performance when there is air-water vapor mixture in the compressor. It may also be pointed out that the first attempt at modeling air-water droplet mixture flow in an axial compressor is presented in Ref. 1 and some modifications to it (to account for different droplet sizes) is presented in Section 4 of this Report. No calculation schemes based on such an analysis have yet been evolved and the models themselves need further improvement.

Regarding upstream effects due to changes in a given stage of the compressor, no attempt at evolving a theory has been undertaken in this Report.

#### 2.1.2. Solution procedure

The solution procedure and a computer program (Purdue University Compressor Performance Program, PUCPP-001) are described in Appendix 1 and Appendix 2 to this report.

#### 2.1.3. Results for typical compressors

Calculations of changes in compressor performance have been performed on two compressors, namely

- (1) N.G.T.E. Reference #109 Compressor (Refs. 1 and 2) and
- (2) An Allison Research Compressor (6 stages) for which data were supplied by Detroit Diesel Division, Indianapolis.

In both cases, we have investigated the effect of evaporation of 2.5% by weight of water at chosen locations in the two compressors. There is no special significance to the value of 2.5% by weight for water that is supposed to undergo evaporation.

It may be pointed out here that the N.G.T.E. compressor (Ref. 2) and the Allison Research Compressor (Ref. 3) differ appreciably in their stage characteristics and stage loading. Calculations performed on these are expected to show the effects of the differences in those parameters.

In the calculations performed on the NGTE compressor, the entry conditions have been assumed to be as follows.

$$T_{o_1} = 485.0K$$

$$P_{o_1} = 1 \text{ atm. (standard)}$$

The temperature of 485.0K has been selected for air as follows: suppose water, at a temperature of 288.0K, is introduced into the air stream at a mass rate of 5.0% of that of air; then at the air stream temperature of 485.0K, all of the water would be able to undergo evaporation. Thus, if the air temperature is 485.0K at entry to the first stage of the compressor, one could expect complete evaporation of 2.5% by weight of water, which is the case investigated here.

Suppose now that it is desired to obtain the evaporation of the same amount of water at entry to the second stage. Then a larger amount of water needs to be ingested at entry into the first stage such that the temperature at entry to the second stage (equal to 485.0K plus the rise in temperature in the first stage) is sufficient to evaporate the required amount of water.

It may be recalled that in these calculations the presence of water (prior to evaporation) is assumed not to affect the performance of the

compressor. Thus, if 2.5% by weight of water is assumed to evaporate at entry to the 4th stage, while the inlet air temperature is 485.0K, even though this would require the presence of water in the first three stages and also in the 4th stage (in the latter because of the presence of unevaporated water), the performance of those stages is assumed to be unaffected by the presence of water.

Calculations have been performed on the N.G.T.E. compressor at different speeds of rotation for six different cases corresponding to the same mass of water, namely 2.5% by weight, undergoing evaporation at entry to each of the six stages of the compressor while the inlet temperature is held constant at 485.0K.

It is important to point out that a part of the calculation has been based on extrapolation of the stage characteristics.

In the case of the Allison compressor, the entry conditions have been assumed to be as follows.

$$T_{o_1} = 533.2R$$

$$P_{o_1} = 1 \text{ atm. (standard)}$$

The temperature of 533.2R corresponds to a chosen test condition and therefore an arbitrarily selected value for the purposes of the calculation illustrated here. With that value of inlet temperature, it is clear that by adjusting the amount of water that is ingested into the compressor at inlet one can obtain evaporation of 2.5% by weight of water at least at entry to certain stages. It can be shown easily, considering

the stage temperature rises in the Allison Research Compressor, that evaporation of 2.5% by weight of water can occur only at entrance to the 4th, 5th and 6th stages when the inlet temperature is 533.2R.

2.1.3.1 The results of calculations on the N.G.T.E. compressor are presented in Figs. 2.1 - 2.3 for three speeds of rotation, namely 9,500, 8,000 and 6,000 R.P.M., respectively.

In each of the Figures, the corrected mass flow is defined as follows.

$$\text{corrected mass flow} = \frac{\dot{m}\sqrt{\theta}}{\delta}$$

where  $\dot{m}$  is in lbm/sec,  $\theta = T_{01}/T_{\text{ref}}$ ,  $\delta = P_{01}/P_{\text{ref}}$ ,  $T_{\text{ref}} = 288\text{K}$ , and  $P_{\text{ref}} = 1 \text{ atm}$ . The corrected speed is defined as follows.

$$\text{corrected speed} = \frac{N}{\sqrt{\theta}}$$

where  $N$  is the speed of rotation per minute.

In each of the figures the performance of the N.G.T.E. compressor is presented in the following manner

- (a) Overall performance of the compressor at the given speed for operation with air only at an inlet temperature of 288°K.



- (b) Overall performance of the compressor at the given speed for operation with air only at an inlet temperature of 485.00°K.
- (c) Overall performance of the compressor at the given speed when the compressor is operated with inlet air temperature of 485.00°K and with sudden evaporation of 2.5% of water at entry to

- (1) stage 1 (curve denoted 1)
- (2) stage 2 (curve denoted 2)
- (3) stage 3 (curve denoted 3)
- (4) stage 4 (curve denoted 4)
- (5) stage 5 (curve denoted 5)
- (6) stage 6 (curve denoted 6)

In examining the figures it is clear that the first three stages seem to fall generally into one category and the latter three stages into a second category. In fact the characteristics of the N.G.T.E. compressor are such that, as shown in Fig. 2.3, it is not possible to estimate the overall performance of the compressor for air-water vapor operation under the given temperature conditions if there is sudden evaporation in stages 1, 2 or 3.

Several observations may be made from those results.

(1) At the higher rotational speeds, evaporation of 2.5% by weight of water, when it occurs at entry to the first three stages, permits only a restricted range of operation in terms of corrected mass flow at entry to compressor.

(2) At the higher rotational speeds, evaporation of 2.5% by weight of water, when it occurs at entry to stages 4, 5 and 6, produces a change

in compressor performance but there is still a wide range of mass flows at which the compressor can be operated.

(3) The differences in performance that are obtained when water is taken to undergo evaporation at different stations in the compressor are a clear indication of the influence of stage loading.

In summary, therefore, the effects of operating a compressor under the condition of evaporation of water at some station in the compressor are dependent upon stage characteristics (drop in efficiency at low mass flows, choking at high mass flows and the mass flow range) and stage loadings.

2.1.3.2. The results of calculations on the Allison Research Compressor are presented in Figs. 2.4 - 2.8 for operation at one speed of rotation and one inlet temperature of 533.2R.

In each of the figures, the corrected mass flow is defined as follows.

$$\begin{aligned}\text{corrected mass flow} &= \frac{\dot{m}\sqrt{\theta}}{\delta} \\ &= \dot{m}\sqrt{T_{01}/T_{\text{ref}}}/(P_{01}/P_{\text{ref}})\end{aligned}$$

where the temperature has been normalized with respect to 518.4R and the pressure has been normalized with respect to 1.0 atmosphere (standard). The corrected speed is defined as follows.

$$\text{corrected speed} = \frac{N}{\sqrt{\theta}} = \frac{N}{\sqrt{T_{01}/T_{\text{ref}}}}$$

The results of calculations on the Allison Research Compressor are presented in Figs. 2.4 - 2.8. In Fig. 2.4 the individual stage

characteristics for the six stages of the compressor are presented at one speed and at air inlet temperature of 533.2°R. These stage characteristics have been employed in all of the subsequent calculations.

In Fig. 2.5, the overall pressure ratio of the Allison Research Compressor is presented for the following cases.

- (a) Operation of the compressor with air only.
- (b) Operation of the compressor with sudden evaporation of 2.5% of water at the stage noted.

In Fig. 2.6 the overall temperature ratio is given for the same conditions as above.

In Fig. 2.7 the overall adiabatic efficiency is given for the same conditions as above.

It will be recalled that only "natural evaporation cases" have been examined here and also at one speed only. Thus, under the conditions of operation, there is no possibility of sudden evaporation of 2.5% of water in stages 1, 2 or 3. The first location where such evaporation can occur is at the exit of the third stage or the entry to the fourth stage.

The influence of stage loading and of the location at which evaporation sets in is clear from those figures.

In Fig. 2.8, in order to illustrate this point further, the performance characteristics of the fourth, fifth and sixth stages are presented, each stage characteristics corresponding to evaporation at the specified stage. Thus, the sixth stage performance is shown when sudden evaporation occurs at entry to the 4th, 5th and 6th stages; and so on. This shows the cumulative effect of various downstream stages when evaporation occurs at one of the earlier stages.

Thus, in Fig. 2.8, the following notation is introduced.

Curves 4/A, 5/A and 6/A: Performance curves of 4th, 5th and 6th stages with 100% air.

Curves 4/4, 5/5 and 6/6: Performance curves of 4th, 5th and 6th stages with 100% air and 2.5% steam when there is sudden evaporation at entrance to the 4th, 5th and 6th stages, respectively.

Curve 5/4: Performance of 5th stage with 100% air and 2.5% steam when there is sudden evaporation at entrance to 4th stage. This curve, therefore, represents the performance of the 5th stage when both 4th and 5th stages operate with 100% air and 2.5% steam.

Curve 6/4: Performance of 6th stage with 100% air and 2.5% steam when there is sudden evaporation at entrance to 4th stage. This curve, therefore, represents the performance of the 6th stage when stages 4, 5 and 6 operate with 100% air and 2.5% steam.

Curve 6/5: Performance of the 6th stage with 100% air and 2.5% steam when there is sudden evaporation at entrance to 5th stage. This curve, therefore, represents the performance of the 6th stage when stages 5 and 6 operate with 100% air and 2.5% steam.

It will be observed that both 5/4 and 6/4 are closer to 5/A and 6/A, respectively, than 5/5 and 6/6. This may be taken as an indication of the cumulative effect of stages 4 and 5 in one case and of stages 4, 5 and 6 in the other case. It just happens in this particular case that



the cumulative effect is "favorable" in that the effect of addition of steam has decreased as the fluid has gone through additional downstream stages. But this must not be assumed to occur universally. In another case with different stage characteristics and stage loading, an entirely different phenomenon may be observed.

## 2.2 Influence of Bleeding or Injection

A simple method of correcting the performance of a compressor is through arranging bleed or injection at a chosen stage of the compressor. Obviously the effect of bleed or injection is felt in the stages downstream of the section where such a correction is applied.

We now consider a multistage compressor operating with two phase flow and suppose the evaporation of water occurs at a certain stage. If the performance of the succeeding stage under those conditions can be brought back to the performance that the compressor would have with air flow only, through injection or bleeding, then the succeeding stages would perform as with air alone and the performance of the compressor would become the same as with air flow under those conditions.

We consider here only those cases where the water content by weight and hence by volume is quite small. Also, we restrict attention to steady state operation sufficiently far from stalling conditions.

When bleeding is arranged it is clear that the blow-off port needs to be located at some stage downstream of the stage where the ingested water is likely to undergo evaporation.

When injection is arranged, one has to decide what air to inject. One of the options -- and this may be the only practical option -- is to divert a small amount of air from the exit plane of the compressor

into the port for injection. In fact this is the case considered in the following analysis.

#### 2.2.1. Problem Formulation

The following assumptions are introduced:

- (1) Same as assumptions (1) under 2.1.1.
- (2) The air that is injected is the air from the exit plane of the compressor when it is operating with evaporated water.

Based on the foregoing assumptions, the problem is formulated as follows. Referring to Fig. 2.9, we consider the performance of a certain stage of the compressor in terms of corrected mass flow at entry to the stage and pressure ratio across the stage at a certain value of corrected speed. Suppose the performance of the stage with air alone is designated by (A) and the performance of the stage with evaporation of a certain amount of water is designated (A + S). It is clear that there is a change in the corrected speed of the stage following evaporation in view of the reduction in temperature that arises during evaporation. Suppose now that the particular stage is operating at point (a) on (A) with air alone and, on evaporation of a certain amount of water at entry to the stage, the performance of the stage shifts to point (1). The problem then is to establish the bleed and/or injection (the latter with the mixture drawn from the exit of the compressor, say) that is required in order to readjust the performance of the stage towards point (a). However point (a) represents a certain corrected mass flow and pressure ratio. In practice, in general, it may be impossible to return the compressor stage that has

begun to operate at point (1) to operate at point (a). In a given case, therefore, it is appropriate to establish whether bleed and/or injection has to be introduced at (1) to move towards (a) and what the effect of the magnitude of bleed and/or injection is at (1).

For the case illustrated in Fig. 2.9 it is clear that bleeding alone is not effective in changing the operating point (1) towards the operating point (a). There are then two other possibilities for adjusting the operating point of the stage, namely injection alone or a combination of bleeding and injection. The latter may become necessary if for various practical reasons one sets limits on the magnitude of injection and bleeding.

#### 2.2.2. Solution Procedure

The solution procedure and the associated computer program (Purdue University Compressor Performance Program, PUCPP-002) are described in Appendix 3 to this report.

The program is intended for recalculating the performance of a stage with bleeding, injection (or air at any desired temperature and pressure) or combinations of those.

One can estimate the nature of the change in mass flow required for a desired change in performance or the change in performance that can be realized with a given change in mass flow. The effect of injection/bleeding is dependent upon the stage loading in addition to change in mass flow.

#### 2.2.3. Results of an Illustrative Case

Investigations have been performed on the 4th stage of the N.G.T.E. #109 compressor.

Assuming air as the working fluid, the performance of the 4th stage of the compressor is established for a range of mass flows at entry to the first stage of the compressor. This is illustrated in Fig. 2.10.

Next, assuming that 2.5% by weight of water has evaporated at entry to the 4th stage the performance of the 4th stage is established again and is also shown in Fig. 2.10.

It may be pointed out here that the performance curves shown in Fig. 2.10 for stage 4 are a consequence of the range of mass flow selected at entry to the 1st stage of the compressor. The available stage characteristics for the 1st, 2nd and 3rd stages, even after some extrapolation, do not permit any larger mass flows to be considered, than used here, at entry to the 1st stage.

Now, calculations were performed to establish the effect of injection of three different mass flows from the exit of the compressor to the entry section of the 4th stage when that stage was operating with 2.5% water evaporated at entry to it. The magnitudes of the injected mass were 2.0%, 5% and 10%. The resulting performance of the 4th stage is again illustrated in Fig. 2.10.

Two considerations to be taken into account in performing such calculations are as follows.

- (1) The effect of injecting a certain percent of compressor exit flow into an upstream stage such as the 4th stage can be established only by iteration since the compressor exit condition itself changes with upstream injection.



(2) From the same reasoning, the compressor exit condition changes when different amounts of exit flow are introduced into an upstream stage such as the 4th stage.

Referring to the operating lines presented in Fig. 2.10, while it appears that, with 5% injection of compressor exit flow, it is possible to come close to the performance characteristic with air alone, there are large differences between the individual operating points with air alone, with evaporation but no injection and with evaporation and injection.

It is important to observe here that the foregoing applies to a particular stage in a given compressor with a chosen magnitude of evaporation at entry to the stage. The same conclusions may not be reached in another case. All that may be said at this stage is that, there are limits to the magnitude of bleeding and/or injection, the stage loading and the location and magnitude of evaporation will determine the nature and extent of the changes that can arise through bleeding and/or injection.

### 2.3 Influence of Location of Water Vaporization

In both of the previous discussions under Sections 2.1 and 2.2, we have emphasized the location of water vaporization in addition to the mass of water evaporated and the stage loading of the stages following the location of vaporization.

In Ref. 3, a test has been reported wherein a slug of water was ingested into a compressor under two different conditions: (1) cold condition and (2) post warm-up condition. It was found that under cold conditions there was no noticeable effect on the engine performance.

However, when the slug of water was ingested after warm-up of the engine, there was both surging of the compressor and after-burner blow-out.

The warm-up condition may be considered as equivalent to a rise in the temperature of ambient air or as equivalent to a shift upstream of the location of water vaporization in a given compressor.

In the alternative, the warm-up condition may have lead to instantaneous differential expansion between the rotor and the casing of the compressor and hence to enlarged clearance and consequent change in performance. At this stage, it is not possible to state positively what the cause for the changed performance is. Nevertheless, the performance of a compressor has been shown here to deteriorate when evaporation of water is advanced upstream in the compressor.

# NGTE. REFERENCE #109 COMPRESSOR

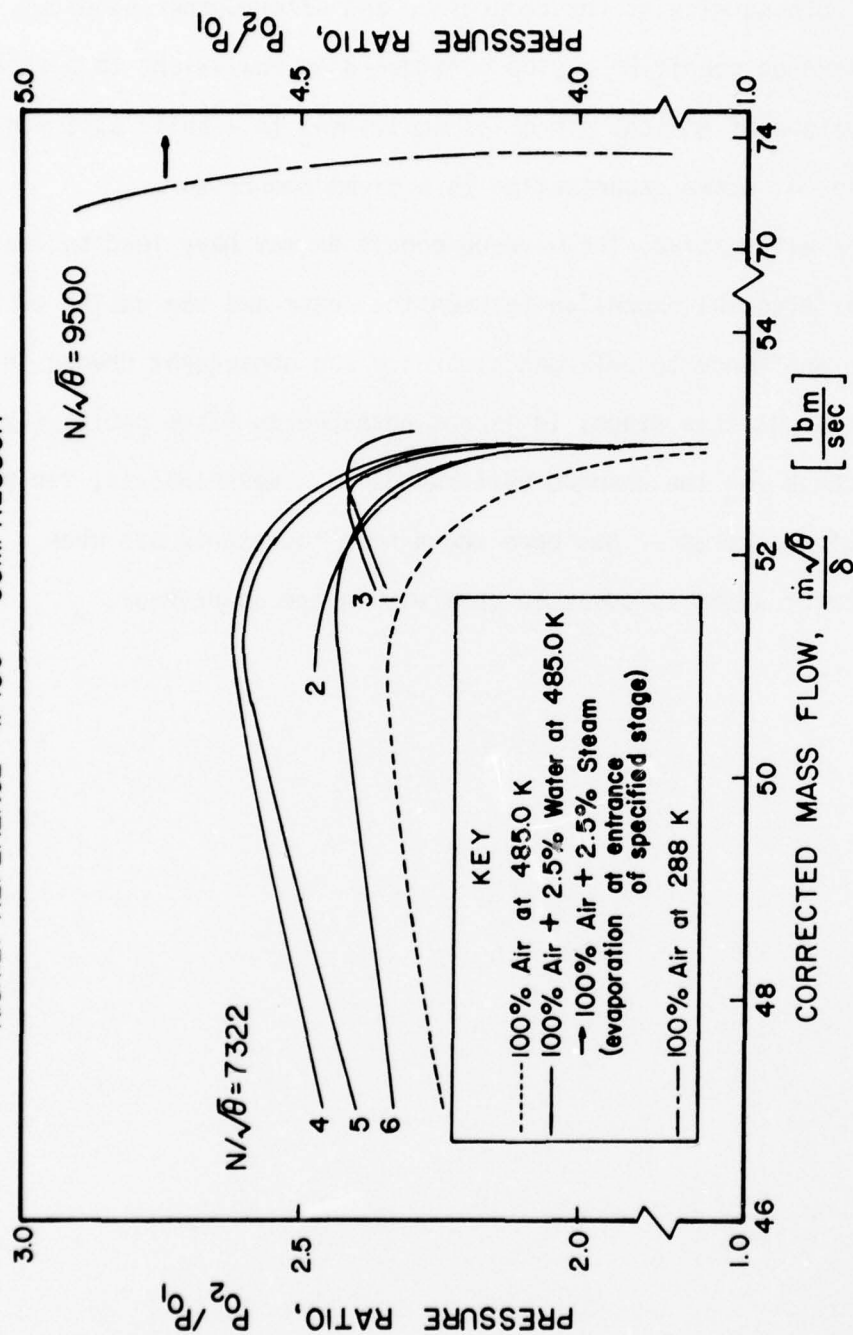


Figure 2.1 Pressure Ratio vs. Mass Flow—9500 RPM

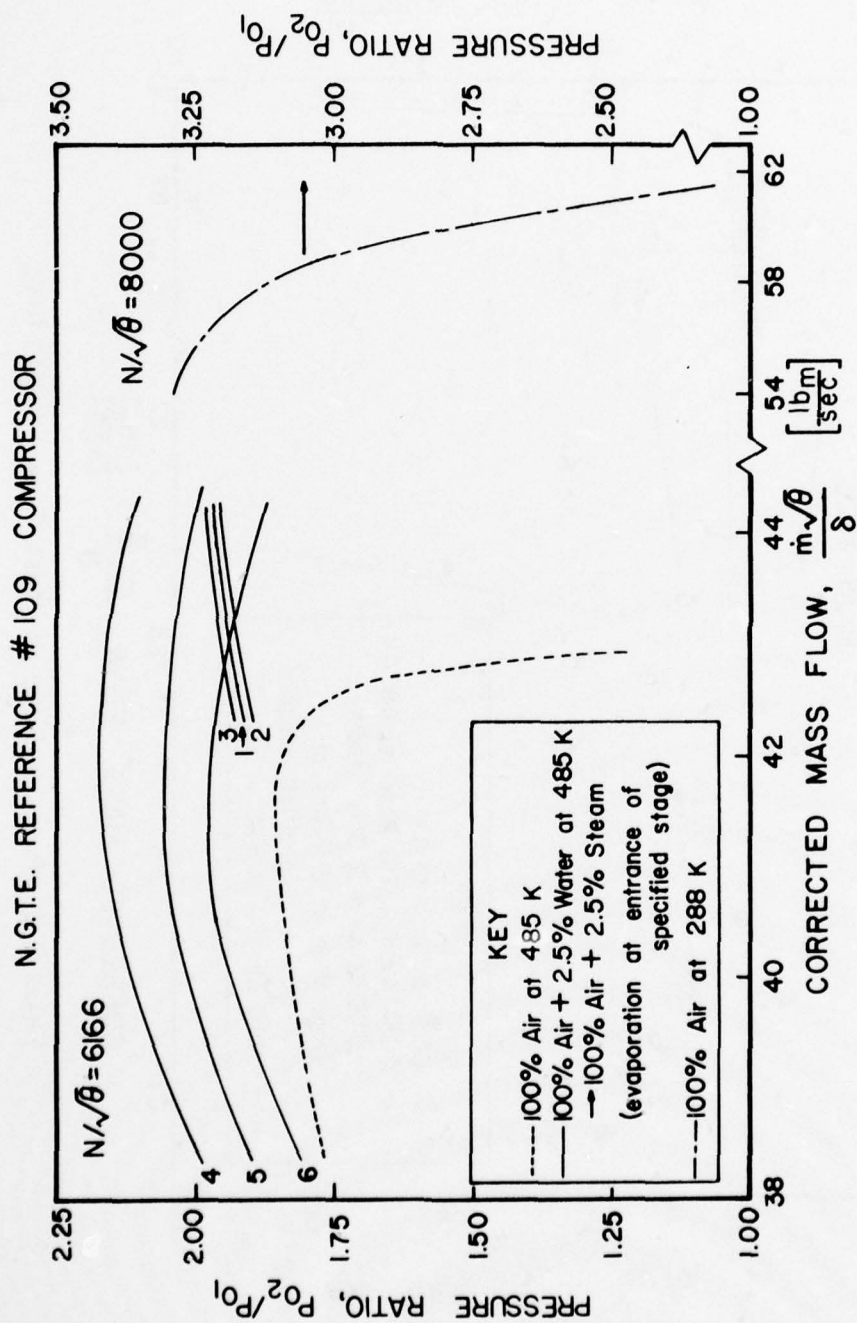


Figure 2.2 Pressure Ratio vs. Mass Flow—8000 RPM



# N.G.T.E. REFERENCE #109 COMPRESSOR

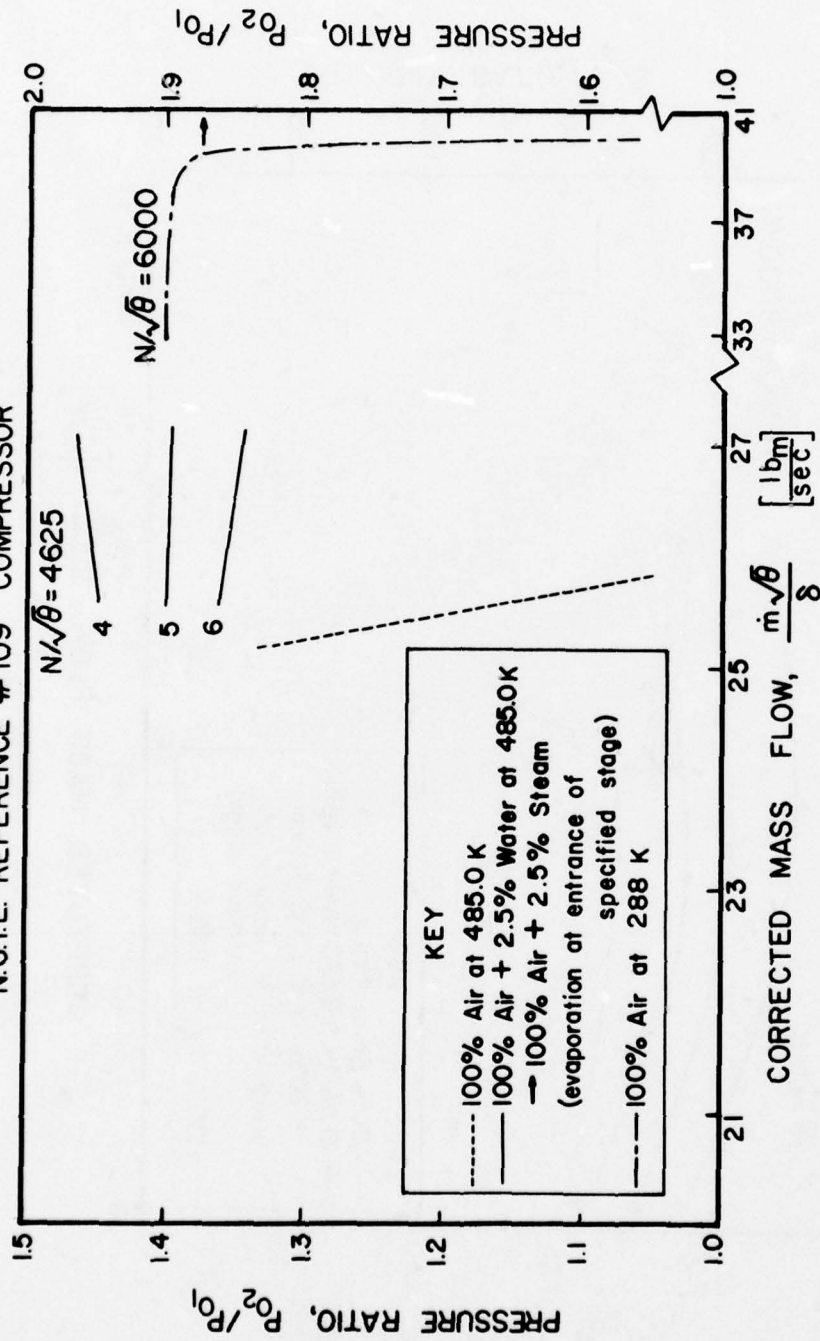


Figure 2.3 Pressure Ratio vs. Mass Flow—6000 RPM

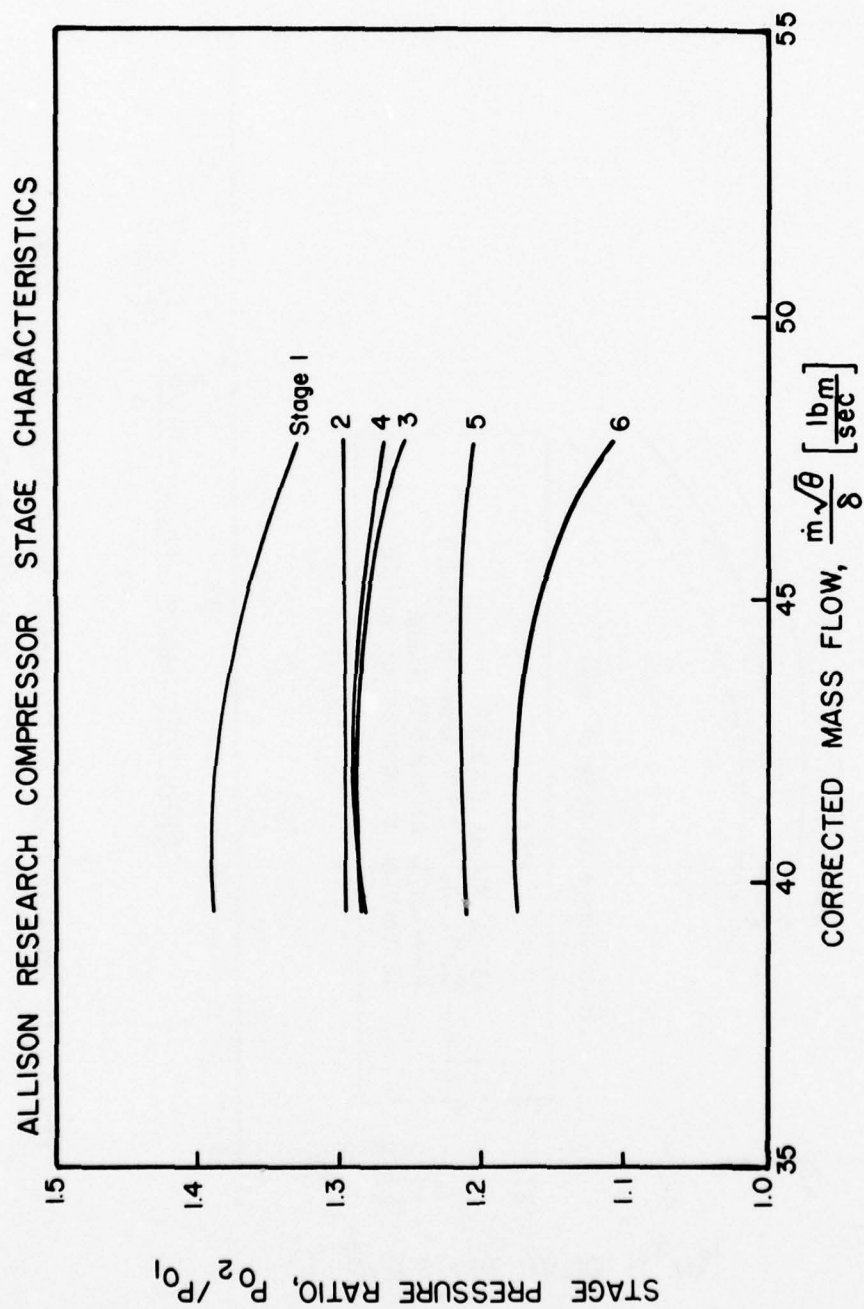


Figure 2.4 Stage Pressure Ratio vs. Corrected Mass Flow—9939 RPM ( $N/\sqrt{\theta}=9800$ )

# ALLISON RESEARCH COMPRESSOR

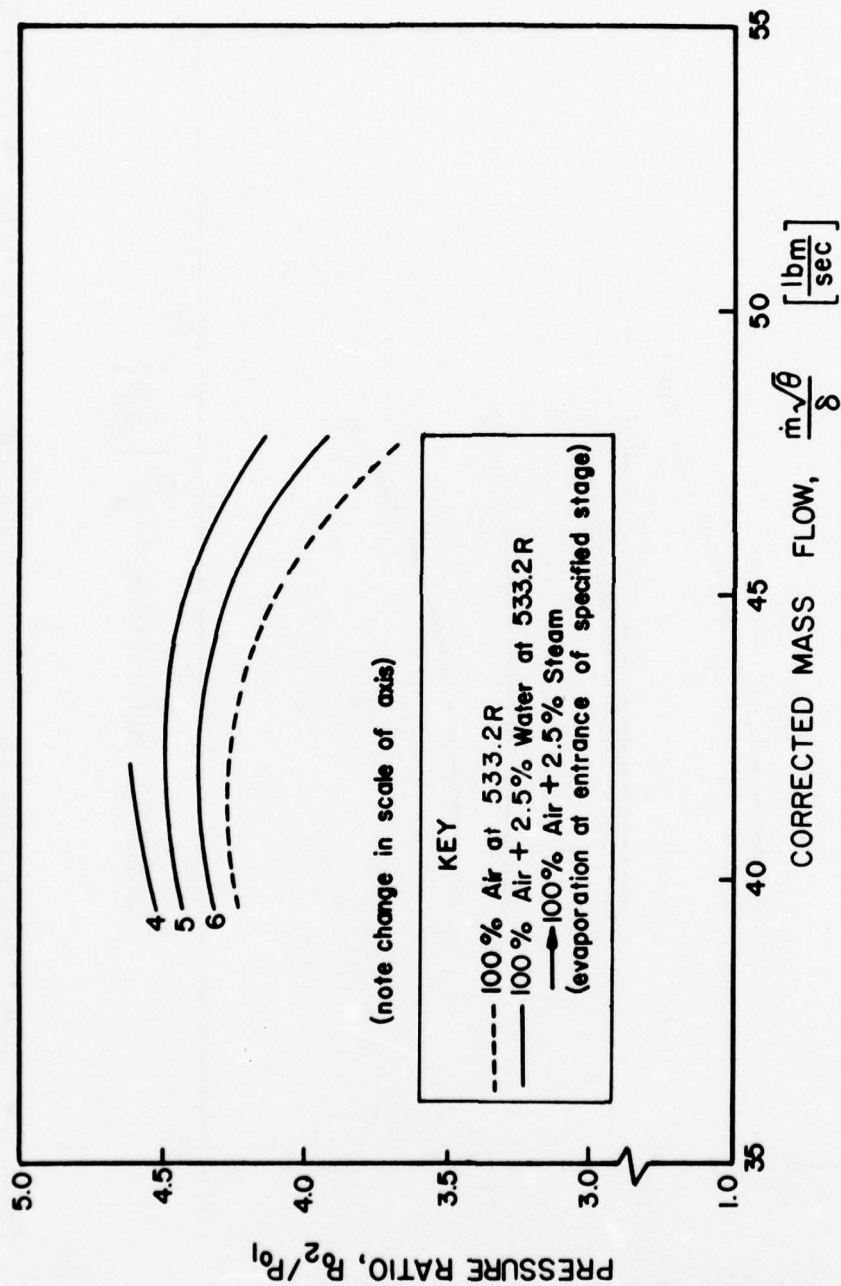


Figure 2.5 Pressure Ratio vs. Corrected Mass Flow—9939 RPM  
( $N/\sqrt{\theta} = 9800$ )

# ALLISON RESEARCH COMPRESSOR

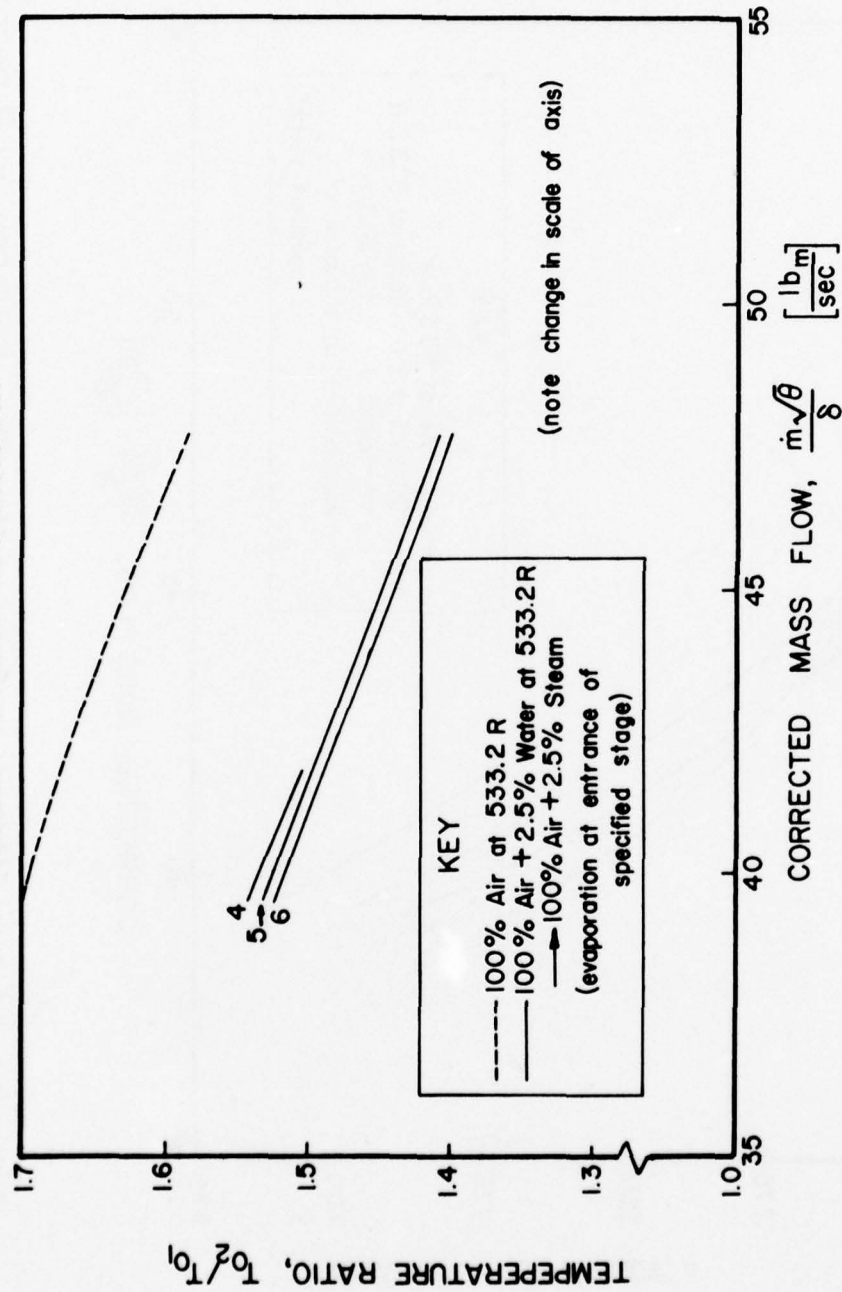


Figure 2.6 Temperature Ratio vs. Mass Flow—9939 RPM ( $N/\sqrt{\theta} = 9800$ )



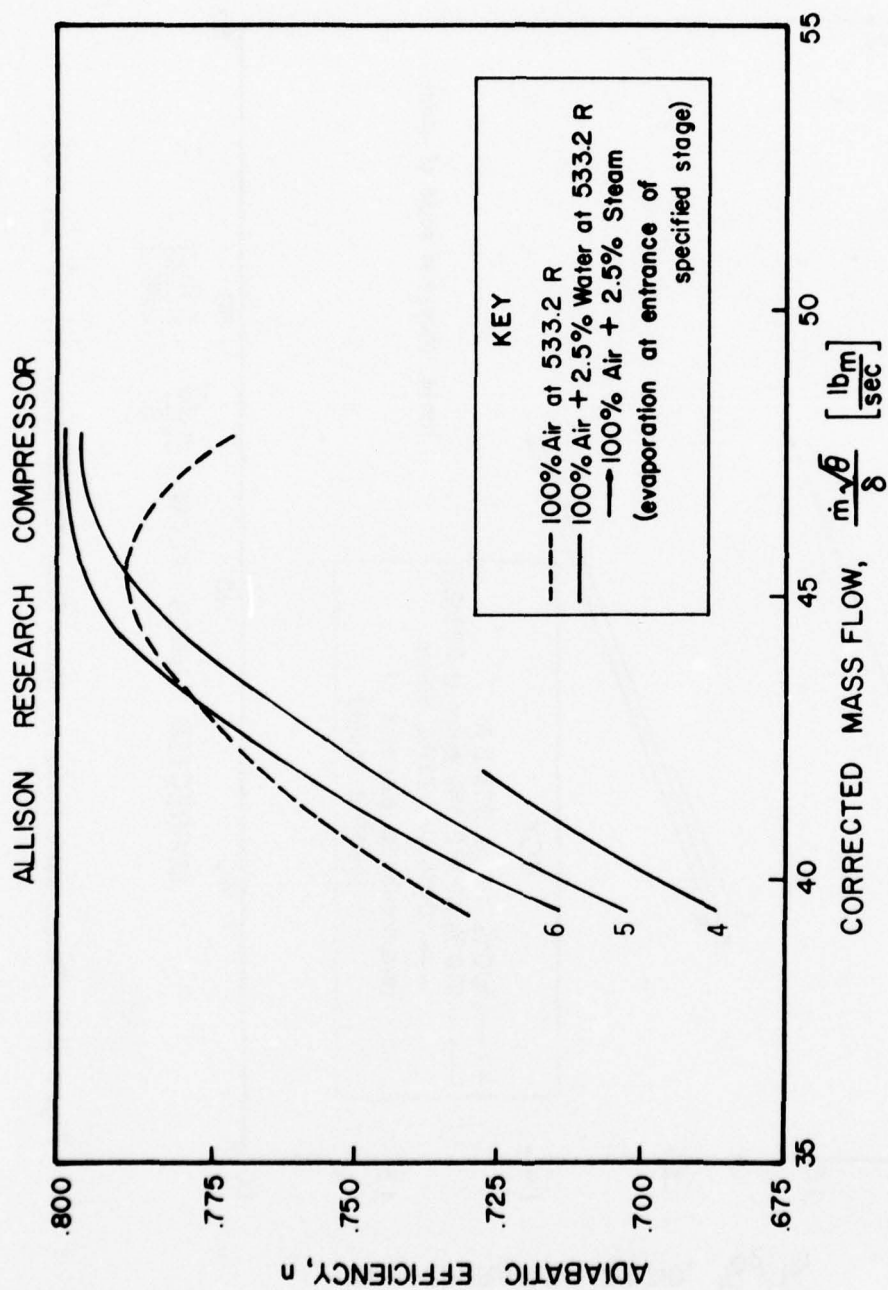


Figure 2.7 Adiabatic Efficiency vs. Corrected Mass Flow — 9939 RPM ( $N\sqrt{\theta} = 9800$ )

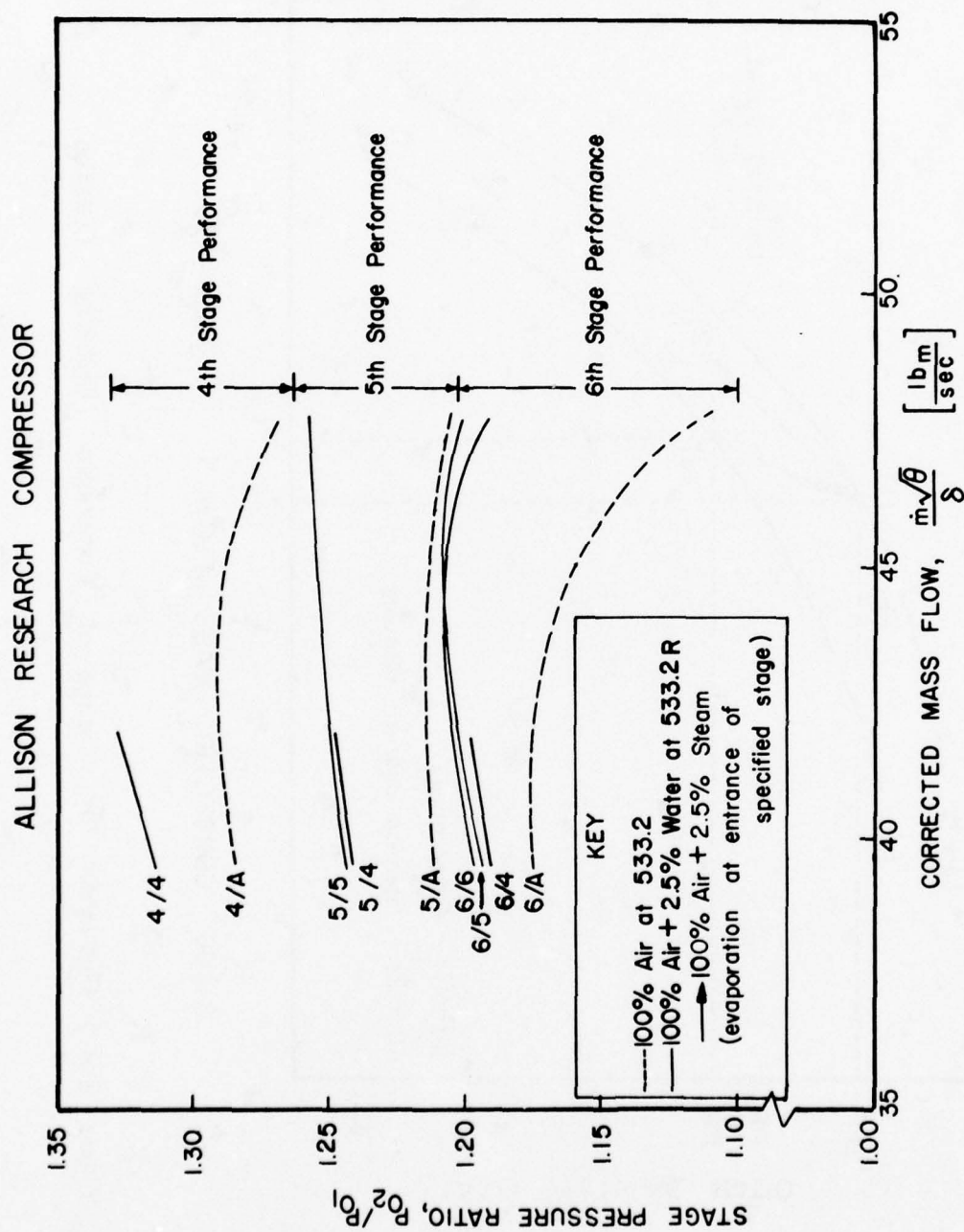
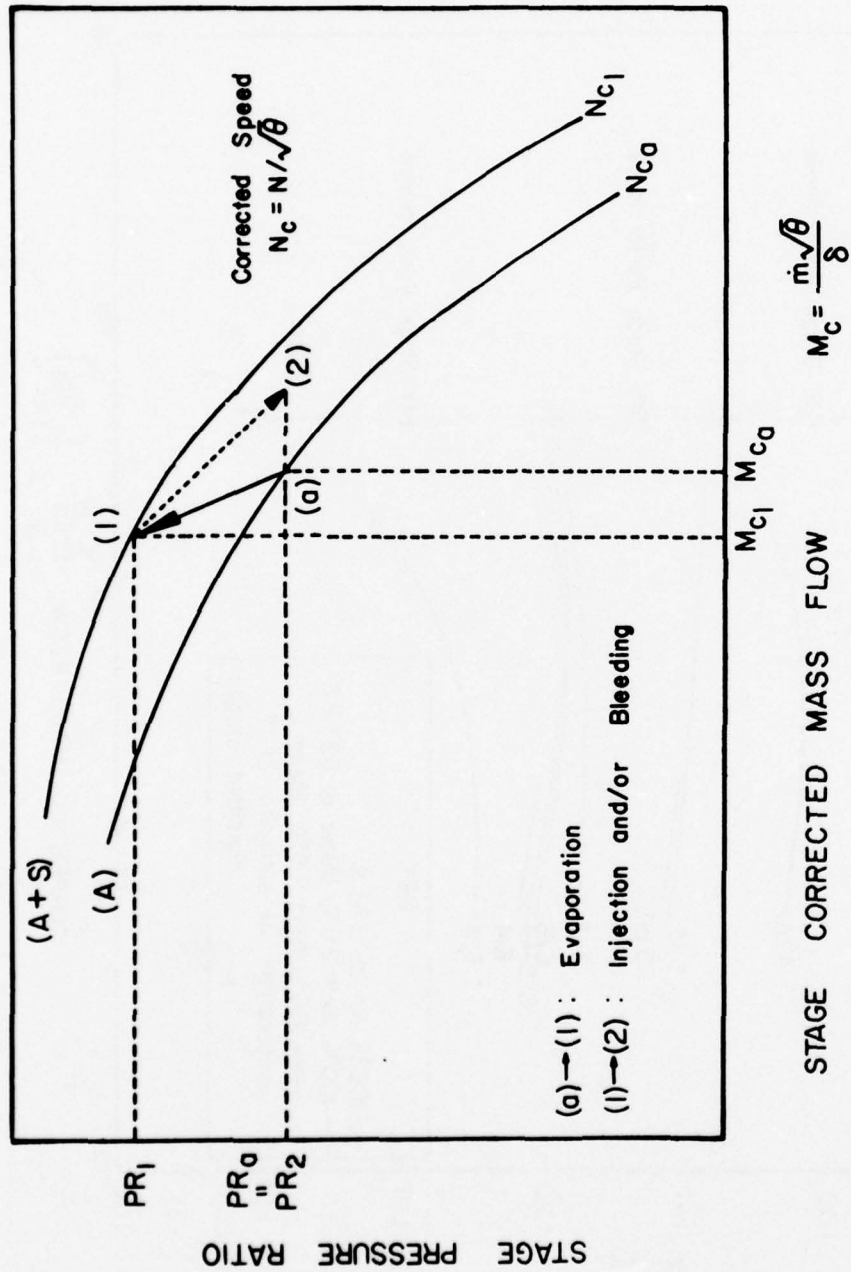


Figure 2.8 Stage Pressure Ratio vs. Mass Flow—9939 RPM ( $N/\sqrt{\theta}=9800$ )



**Figure 2.9** Illustration for Change of Compressor Operating Condition

# 4th STAGE PERFORMANCE—N.G.T.E. REFERENCE #109 COMPRESSOR

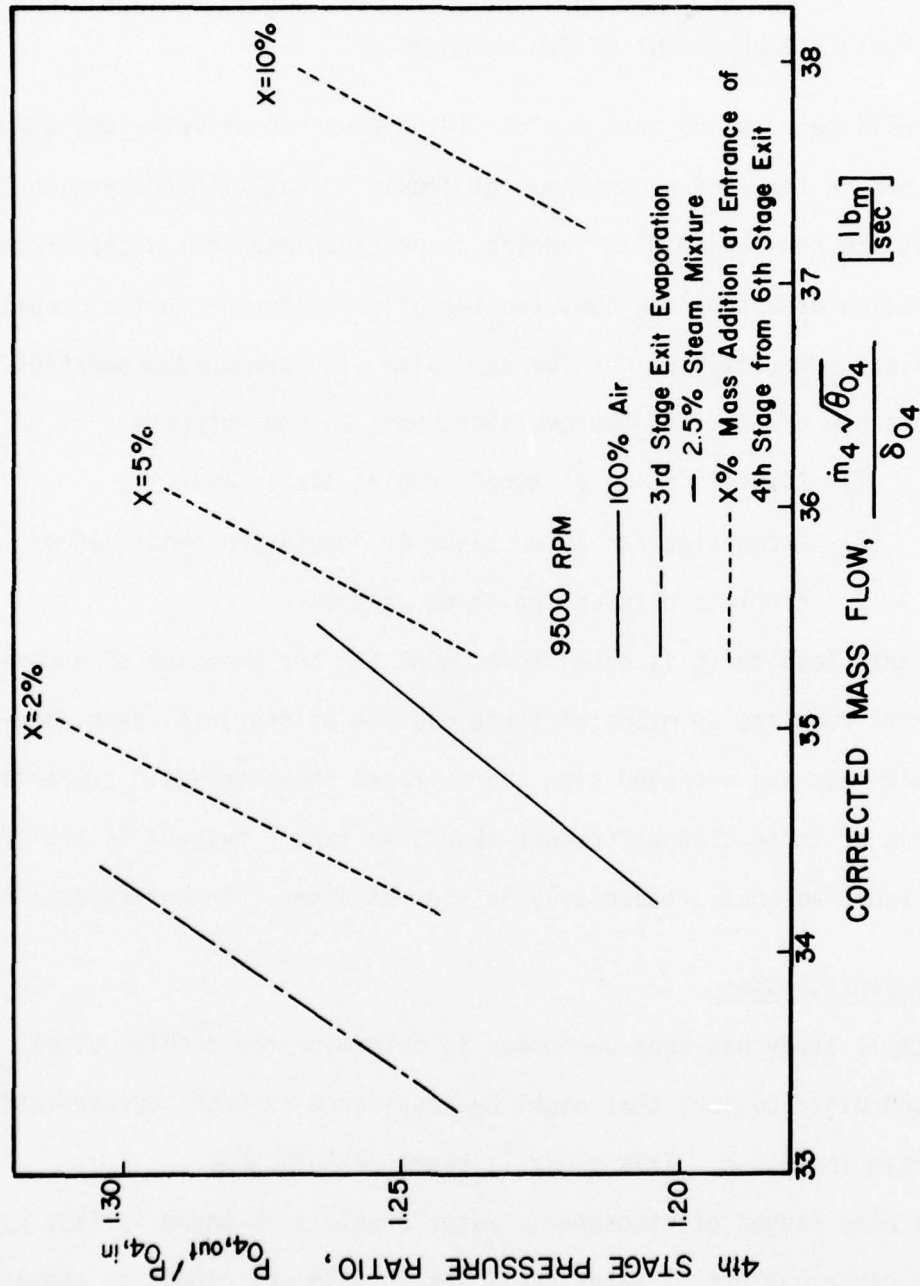


Figure 2.10 Stage Pressure Ratio vs. Corrected Mass Flow



### SECTION III

#### FURTHER DEVELOPMENT OF TWO-PHASE MODEL

It will be recalled that in Ref. 1 the presence of water was accounted for in terms of a continuum of droplets, but with the assumption that the non-random distribution of droplets does not necessitate the inclusion of a separate contribution of pressure due to the droplets. The droplets were assigned an averaged size with averaged properties.

It is now proposed to improve that model in two respects.

- (1) Identification of three droplet sizes; and
- (2) Accounting for interchange of population densities of droplets between the three classes.

In this fashion it is hoped to account for the presence of a variety of discrete droplets by means of three classes of droplets, each associated with its own averaged size and averaged characteristic properties. The choice of three classes (rather than some larger number) is arbitrary but may prove adequate, especially in view of other inherent uncertainties.

#### 3.1. Droplet Classes

A small study has been performed to determine the droplet sizes (range and distribution) that might be considered as most representative during rain ingestion. This study is based on Refs. 4-6.

The size ranges of atmospheric water droplets is shown in Fig. 3.1. The size distributions of water particles in rain and clouds is shown in Figs. 3.2 and 3.3, respectively. The values indicated in those figures should be considered as typical, noting that there can be wide

discrepancies in the estimates of such distributions. Utilizing the information in Figs. 3.2 and 3.3, a particular classification of water particles has been chosen, as shown in Fig. 3.4, for use in this report.

It will be observed that three ranges of particle sizes are selected as follows.

- (1) drizzle, ranging from 5 to 30 microns;
- (2) small droplets, designated Class 1 and ranging from 100 to 800 microns; and
- (3) large droplets, designated Class 2.

Those three classes are further characterized as follows.

(a) In the case of drizzle, it is assumed that it is a continuum moving with the air with no viscous interaction identified between the particles and the air. These small particles are further assumed to introduce no pressure on the system.

(b) In the case of the other two sizes of particles it is assumed that each class may again be treated as a continuum of some assigned, averaged size particles but each class moving with its own characteristic velocity. In view of the larger sizes of these particles, viscous interaction is taken into account between the particles and air. Again it is assumed that neither of these classes of particles exerts any pressure force on the system.

(c) Finally, in the case of the two larger size particles, it is assumed that coalescence and disintegration of droplets can take place according to accepted criteria for such processes. However, the small droplets, Class 1, when they coalesce (among themselves or with Class 2

droplets) are always expected to yield Class 2 droplets. Similarly the large droplets, Class 2, when they break-up are always expected to yield Class 1 droplets. In other words, Class 1 droplets cannot coalesce into other Class 1 droplets, nor can Class 2 droplets break-up into other Class 2 droplets.

### 3.2. Model for Coalescence and Break Up

The interaction between Class 1 and Class 2 droplets is expected to arise either through droplet coalescence or droplet break up. The interaction model is similar to that employed in elementary molecular theory (Ref. 7).

The model is developed as follows.

(a) The droplets in Classes 1 and 2 each lie between the diameters  $D_{i \min}$  and  $D_{i \max}$  ( $i = 1, 2$ ), but class can be represented by a characteristic size  $\bar{D}_i$ .

(b) Suppose  $N_i$  is the number of droplets of range  $i$  per unit volume. A distribution function  $f_i$  is then introduced such that

$$dN_i = N_i f_i dD \quad (1)$$

where  $dN_i$  is the number of droplets of type  $i$  per unit volume in the diameter range  $D_i$  and  $D_i + dD$ .

(c) The velocity of all of the droplets of type  $i$  is denoted by  $u_i$ .

(d) Coalescence is supposed to occur by collisions between droplets of the two Classes 1 and 2, but not by collisions in Class 1 or 2. Break up is assumed to occur when a critical value of Weber number is reached.

### 3.2.1. Coalescence model

A notion of a collision time  $\Delta\tau$  is introduced, for example  $\Delta\tau$  can be related to the half period of a free oscillation of a spherical droplet. Then the collision volume or the volume traversed by droplets during a collision can be expressed by the relation,

$$\Delta V = \frac{\pi}{4} (\bar{D}_1 + \bar{D}_2)^2 (u_1 - u_2) \Delta\tau. \quad (2)$$

Let  $n_1$  be the number of collisions experienced by particles of Class 2, which is the same as that experienced by particles of Class 1, in the volume  $\Delta V$ . Then, it follows that

$$n_1 = N_1 \Delta\tau \int \frac{\pi}{4} (\bar{D}_1 + \bar{D}_2)^2 (u_1 - u_2) f_1 dD \quad (3)$$

We will assume here that the mean droplet size, the mean and standard deviations are such that, when the droplets obey normal distribution, nearly all of the collisions will have been accounted for in the analysis.

Now, the total number of collisions per unit time can be written as

$$n = N_1 / \Delta\tau \quad \text{for } N_2 > \frac{1}{\Delta V} \quad (4)$$

$$= \frac{N_1}{\Delta\tau} \cdot \frac{N_2}{(1/\Delta V)} \quad \text{for } N_2 \leq \frac{1}{\Delta V} \quad (5)$$

or, in general, as

$$n = RN_2 / \Delta\tau \quad \text{where } R = N_1 / N_2 \text{ for } N_2 > 1/\Delta V \quad (6)$$

$$R = N_1 \cdot \Delta V \text{ for } N_2 \leq 1/\Delta V$$

We can then express the loss rate of mass of type 1 droplets per unit volume of mixture due to collisions in the following form.

$$\dot{m}_{C1} = \frac{RN_2 \rho_1 \pi}{6\Delta\tau} \int D^3 f_1 dD \quad (7)$$



It may be pointed out that the loss rate of type 1 droplets is equal to the gain rate of type 2 droplets.

Let  $\alpha_i$  be the bulk volume concentration of type  $i$  droplets in the mixture. Then

$$\alpha_i = \frac{1}{6} \pi N_i \int D_i^3 f_i dD \quad (8)$$

The mass density of droplets of type  $i$  can then be written as follows.

$$\rho_i = \alpha_i \rho_\ell \quad (9)$$

where  $\rho_\ell$  is the density of the liquid.

Finally, it is necessary to identify the number of droplets of each class, say  $\dot{m}_{Gi}$ , that would become created (due to condensation) or loss (due to evaporation) per unit volume of mixture. We can define the mass of such droplets as follows.

$$\dot{m}_{Gi} = (\dot{m}_{ci} - \dot{m}_{ei}) \quad (10)$$

where the subscripts  $c$  and  $e$  refer to condensation and evaporation respectively.

On a continuum basis, therefore, one has to examine the balance between  $\rho_i$ ,  $\dot{m}_{ci}$  and  $\dot{m}_{Gi}$ . This however does not take into account the break up of droplets, which will be discussed in the following section.

### 3.2.2. Break-up model

In order to account for droplet break-up, we introduce the notion of Weber number,  $We$ , defined by

$$We = \frac{p \cdot 2D}{\sigma} \quad (11)$$

where  $p$  and  $\sigma$  are the environmental pressure acting on the droplet and surface tension, respectively.

We assume that only droplets of Class 2 can undergo break up.

The environmental pressure acting on the droplet can be written as the sum of aerodynamic force and collisional force,

$$p_{\text{aero}} = \rho_g \frac{(u_g - u_\ell)^2}{2} \quad (12)$$

and

$$p_{\text{coll}} = \frac{F_{\text{coll}}}{A^*} \quad (13)$$

where

$$F_{\text{coll}} = \frac{1}{\Delta\tau} \bar{m}_1 (u_1 - u_2) \eta_c \quad (14)$$

and  $\bar{m}_1$  is the average mass of a droplet of Class 1 and  $\eta_c$  is the "efficiency" of collisions. The area normal to  $F_{\text{coll}}$  is given by  $A^*$ .

Now, suppose that the number of droplets breaking up per unit volume is  $N_b$ . Then, introducing an arbitrary constant  $B$ ,

$$N_b = A \exp (B We) \quad (15)$$

One can adopt the following boundary conditions for Eq. 4.12, namely

$$N_b = N_{b0} \quad \text{at} \quad We = We (\text{minimum})$$

and

$$N_b = 1 \quad \text{at} \quad We = We (\text{critical})$$

Thus, it follows that

$$N_b = \frac{N_{bo} + \exp [We/(We_{min} - We_{cr})]}{N_{bo} + \exp [We_{cr}/(We_{min} - We_{cr})]} \quad (16)$$

The total number of droplets generated per unit volume per unit time due to break up is given by

$$N_{1b} = \frac{K}{\Delta \tau} \int N_b V dN \quad (17)$$

where  $K = 0,1$  for  $We < We_{min}$  and  $We \geq We_{cr}$ . Thus, the gain of mass of type 1 droplets due to a break up of type 2 droplets is given by  $N_{1b} \bar{m}_1$ . This of course is equal to the loss of mass of type 2 droplets due to break up.

We can now conclude that there should be a balance between  $\rho_i$ ,  $\dot{m}_{Gi}$ ,  $\dot{m}_{Ci}$  and  $\dot{m}_{Bi}$  taking account of condensation and evaporation, coalescence and break up.

### 3.3. Mass, Momentum and Energy of Liquid Phase

As stated earlier, the part of the liquid phase that is in the form of drizzle is assumed to behave as part of the gas phase. It will undergo evaporation when conditions are right but there is no viscous interaction between the drizzle particles and air, and no interaction among the drizzle droplets.

In essence, therefore, only the Class 1 and Class 2 liquid particles enter into the discussion of the liquid phase.

### 3.3.1. Mass balance equation

The mass balance equation for Class  $i$  droplets may be set up as follows.

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial}{\partial x_i} (\rho_i u_{ii}) = \dot{m}_{Gi} + \dot{m}_{Ci} + \dot{m}_{Bi} \quad (18)$$

### 3.3.2. Momentum balance equation

In addition to the forces taken into account in setting up the momentum balance equation for the liquid phase in Ref. 1, we have to take into account the following forces when two classes of droplets are introduced.

- (1) momentum change due to mass change of each class of droplets;  
and
- (2) momentum change because of interaction between the two classes of droplets.

The latter arises because of each class of droplets being treated as a continuum and the interaction between the two droplet continua. We shall denote this by  $\psi_{Ii}$ .

### 3.2.3. Energy balance equation

In addition to the energy components taken into account in Ref. 1, we have to include the following energy components.

- (1) Energy change due to mass change of each class of droplets;
- (2) Energy required in connection with forces  $\psi_{Ii}$ ; and
- (3) Energy required for the change of droplet surface area during formation of new droplets.



Regarding (iii), if  $E_{Si}$  is the energy involved, we may write this as

$$E_{Si} = \frac{d}{dt} [\sigma N_2 \int \pi D^2 f_2 dD] \quad (19)$$

### 3.4. Equations of Motion

The equations of motion are presented in Appendix . The manner in which they are presented is the same as in Ref. 1, namely,

- I. Three dimensional equations of motion in intrinsic coordinates.
- II. Axisymmetric equations of motion in intrinsic coordinates.
- III. Three dimensional equations of motion in cylindrical coordinates.
- IV. Axisymmetric equations of motion in cylindrical coordinates.

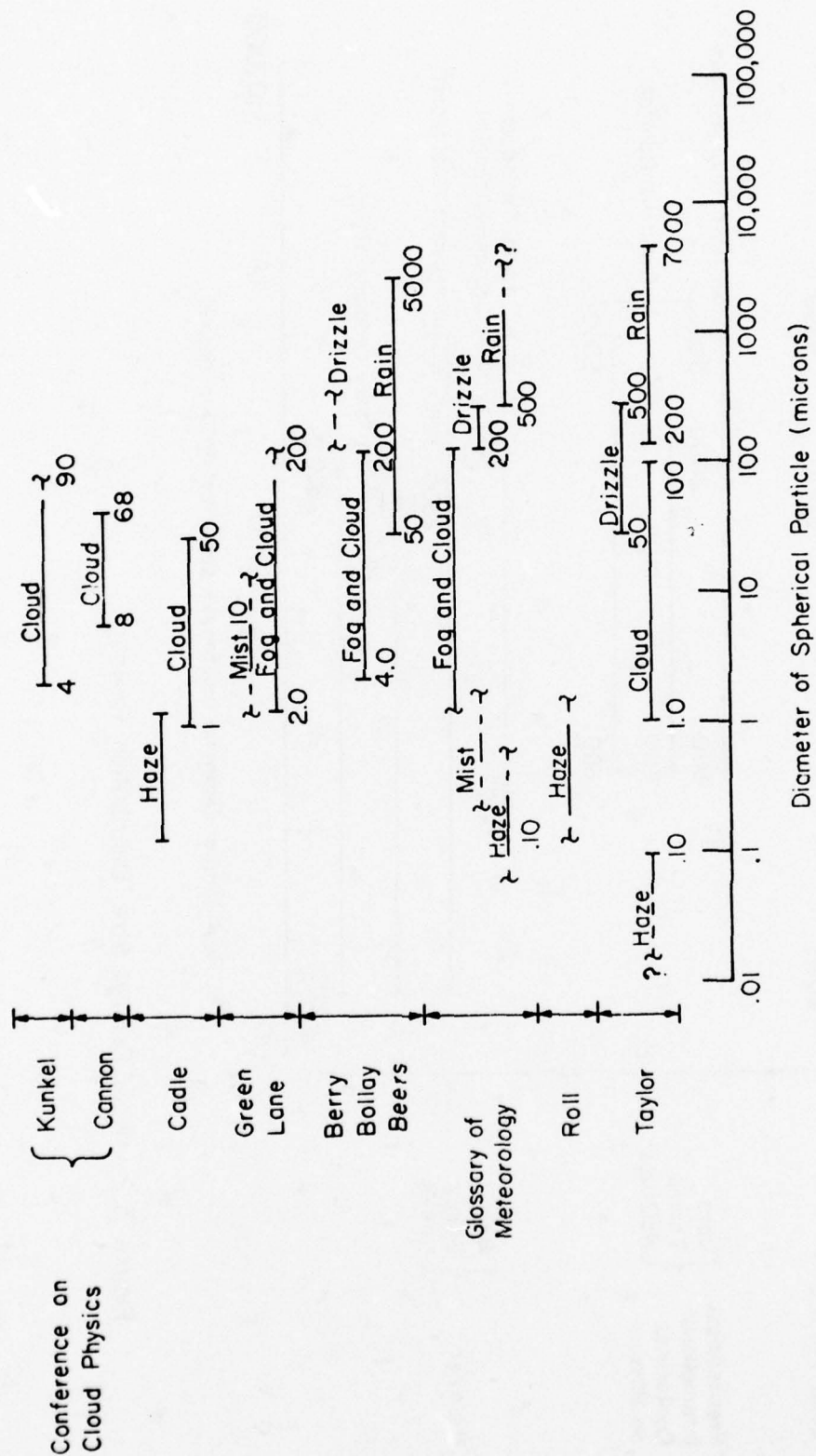


Figure 3.1 Atmospheric Particle Size Ranges

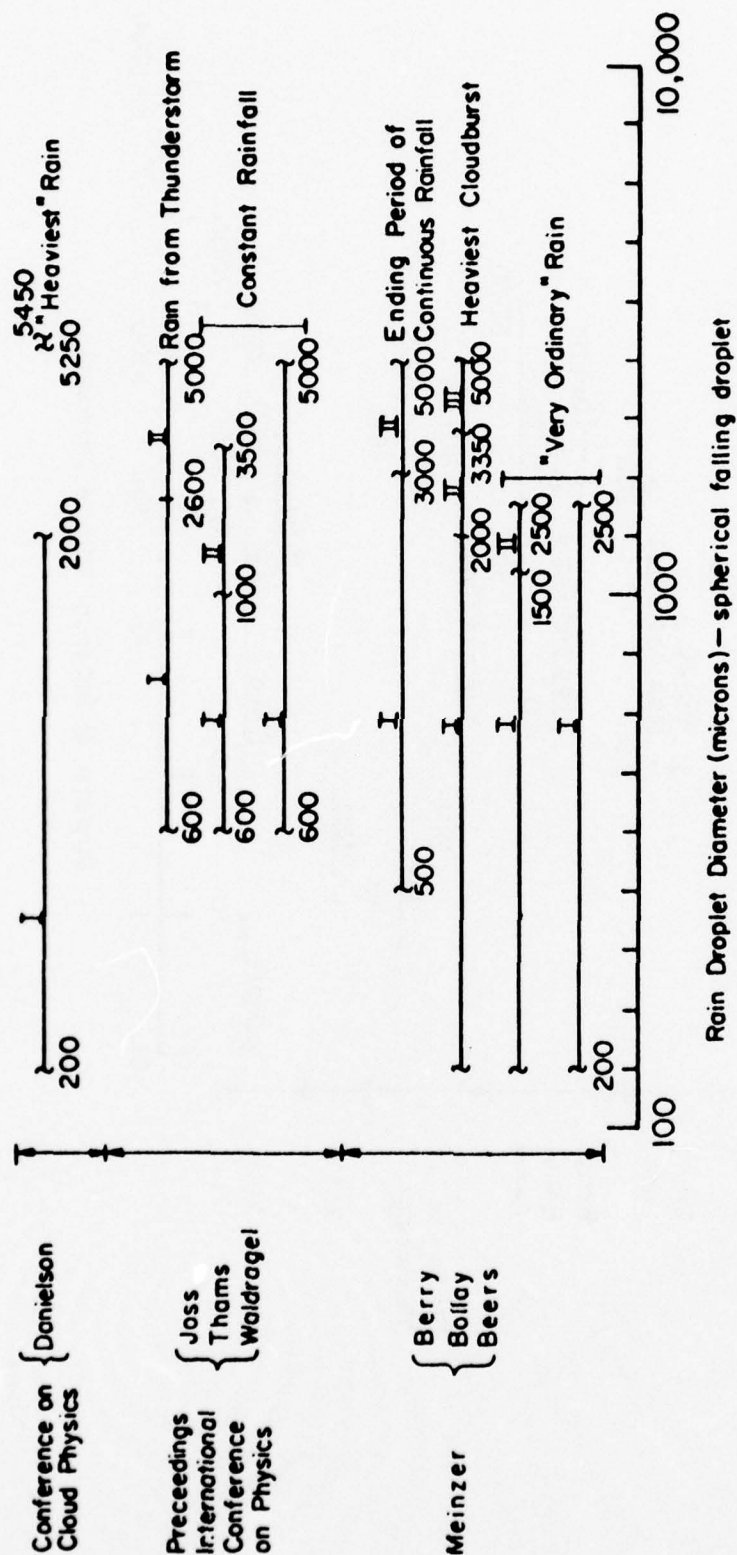


Figure 3.2 Rain Droplet Size Distribution Ranges

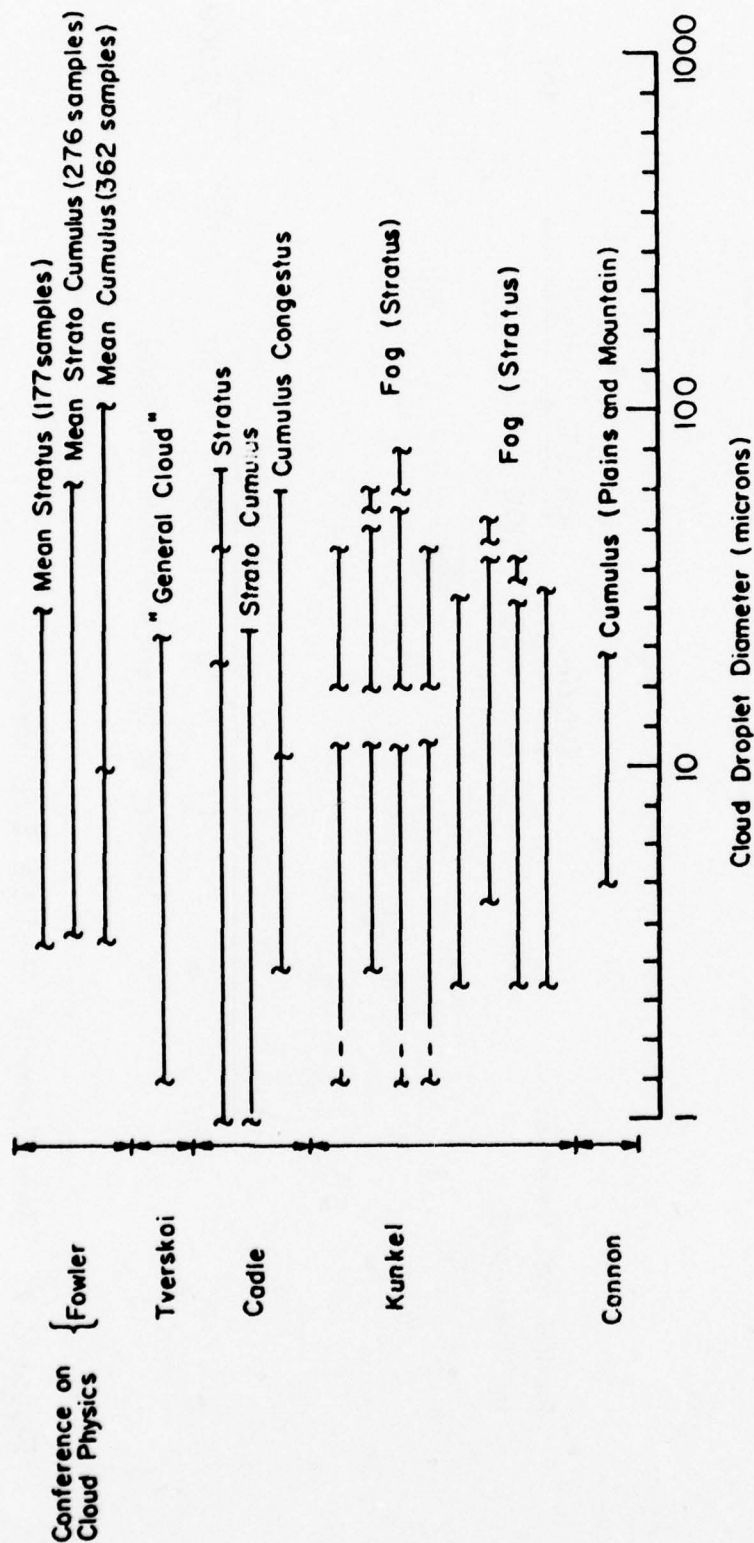


Figure 3.3 Cloud Droplet Size Distribution Ranges



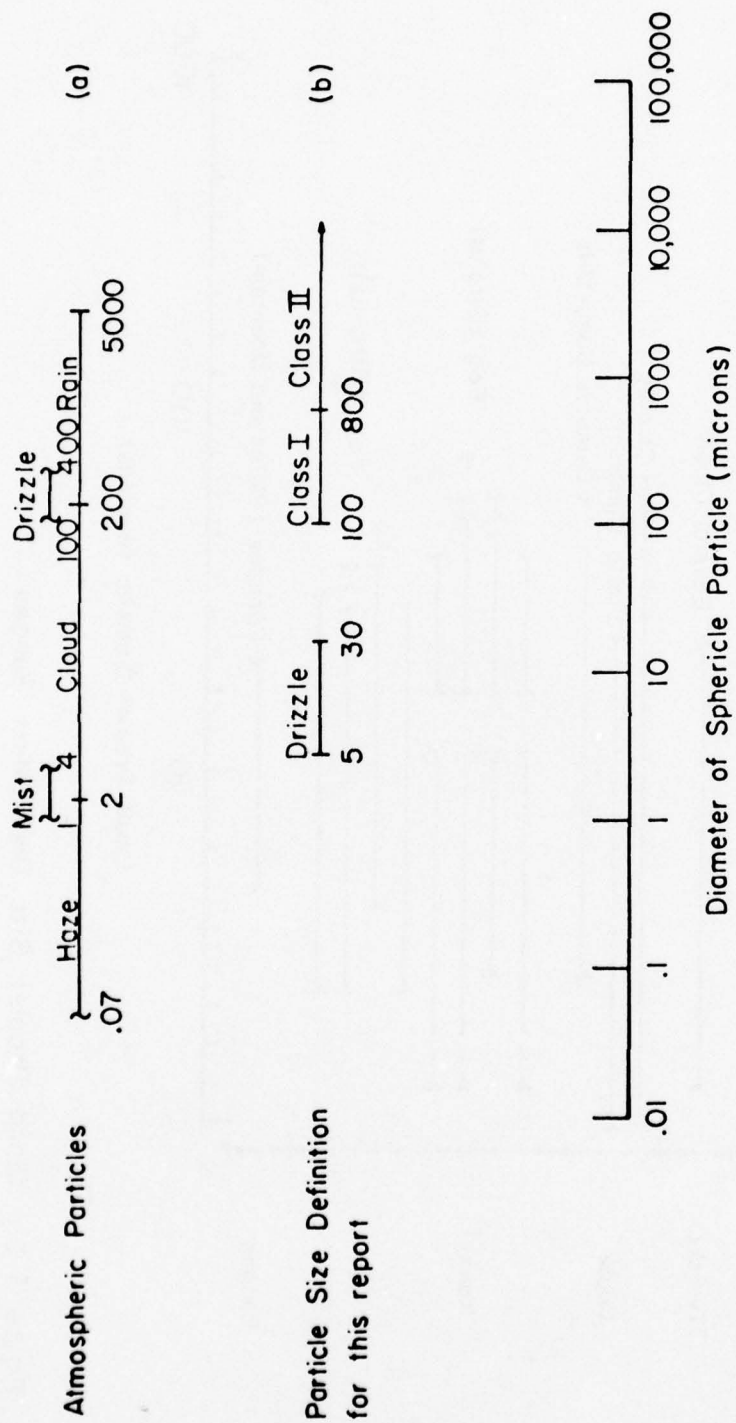


Figure 3.4 "Best Estimate" Particle Size Ranges

SECTION IV  
SOME MODIFICATIONS TO THE  
COMPRESSOR PROGRAM UDO-300

The original program (Ref. 8) has been developed for air flow through a compressor. In order to make it applicable to air-water mixture flow, it is essential to incorporate various modifications. Two of such modifications have been developed to-date and they pertain to the following.

- (1) Need for taking into account variable specific heats and molecular weight along the flow path for the gas phase of the mixture; and
- (2) Need for taking into account variable acoustic velocity for the air-water mixture along the flow path.

4.1 Variable Specific Heats and Molecular Weight

The gas phase in the compressor is assumed to be a mixture of perfect gases. Any water vapor generated due to evaporation of water is assumed to mix instantaneously and homogeneously with the surrounding air.

The specific heat at constant pressure for the gas phase is thus a function of the local temperature.

The molecular weight for the gas phase is a function of the mass fractions of air and water vapor (or steam).

The changes required in the FUNCTION subprograms of UDO-300 are described in Appendix 5.

4.2 Acoustic Velocity in 2-phase Flow

As stated in Ref. 1, acoustic velocity in a two-phase mixture (with one discrete phase) can be defined as follows.

$$c_m = [\{(1 - \sigma_v)\rho_g + \sigma_v\rho_p\} \cdot \{\frac{(1 - \sigma_v)}{\rho_g c_g^2} + \frac{\sigma_v}{\rho_p c_p^2}\}]^{1/2} \quad (20)$$

where  $c_m$  is the mixture sonic velocity,  $\sigma_v$ , the (particulate) liquid volume fraction,  $\rho$ , the density and ( )<sub>g</sub> and ( )<sub>p</sub> refer to the gas and liquid phase respectively. That relation does not include the effect of size distribution of water drops. In order to account for that and also the nonlinear interactions between attenuation and scatter of sound, empirical corrections can be introduced.

Details of the modifications to the FUNCTION subprograms are given in Appendix 5.

## SECTION V

### DISCUSSION

The Report describes five aspects of investigations in the subject of water ingestion in axial flow compressors.

- (1) Analysis of performance of selected compressors with water ingestion.
- (2) Determination of the influence of blow-off and injection into a compressor operating with air-water mixture.
- (3) Development of a model for an engine operating with water ingestion into the compressor.
- (4) Formulation of flow equations for an axial flow compressor operating with air-water mixture accounting for several classes of droplets and such processes as droplet coalescence, break-up and evaporation.
- (5) Some simple modifications to the UDO-300 program to account for variable molecular weight, specific heats and acoustic velocity.

#### 5.1 Conclusions

- (1) The precise effects of the presence of water and of its evaporation on compressor characteristics and surging depend upon  
(a) the stage loading, (b) the change in the performance due to presence of water, (c) the location of evaporation, and  
(d) the changes introduced in the compressor, locally and upstream, due to the various processes associated with water ingestion and evaporation. In an approximate analysis where



- (a) the presence of water in liquid form has been assumed to have little effect on compressor performance, (b) a finite quantity of water is expected to undergo sudden evaporation at predetermined stages and (c) upstream effects induced by changes further downstream are neglected, it has been shown that (a) the location of evaporation has a determining effect on the deterioration of the overall performance of the compressor and (b) such deterioration of the compressor depends upon the stage loading and characteristics of the compressor.
- (2) In general, it is extremely difficult in practice to correct the performance of a compressor that is operating with water evaporated in it (at some intermediate stage) by means of blow-off or injection. In selected compressors, it has been found that the effects of evaporation of even 2.5% by weight of water cannot be rectified by injection of 5% by weight of air flow at practicable temperature and pressure.

## 5.2 Possible Directions for Further Studies

- (1) There is scope for improving the one-dimensional analysis currently employed to assess the performance of selected compressors operating with air-water mixture flow.
- (2) The modeling of overall engine performance under nonsteady state conditions when there is water ingestion is of crucial importance in understanding the effects of water ingestion on controls.
- (3) The flow equations need to be reformulated in the regions of blade and casing surfaces and also to account for mixture compressibility effects.

- (4) Most important of such further studies is the one that will provide proper and adequate experimental information on the effects of water ingestion into isolated compressors and engines. Instrumentation and the development of observation techniques for such studies require in most cases further development.

## APPENDIX A

### PERFORMANCE PARAMETERS

The performance of a compressor may be expressed in terms of the following parameters on a one-dimensional flow basis.

- 1)  $V_a/a$
- 2)  $U/a$  and
- 3)  $P_{0e}/P_{0i}$
- 4)  $\eta_c$

where  $V$ ,  $a$ ,  $U$ , and  $P_0$  and  $\eta_c$  represent the axial velocity, acoustic speed, rotational speed, total pressure and efficiency, and  $( )_i$  and  $( )_e$  refer to the inlet and exit conditions of a compressor. If a single stage of a multi-stage compressor is considered, one can again express the performance on the basis of the same parameters for the stage under consideration.

In general, the parameter  $V_a/a$  may be expressed in terms of the following relation.

$$\frac{V_a}{a} = \frac{\dot{m}\sqrt{T_0}}{P_0} \sqrt{\frac{1}{\gamma \cdot MW}} \left[ 1 + \frac{\gamma - 1}{2} M^2 \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad (A-1)$$

If a compressor, designed originally for operation with air (a), is expected to be operated with another gas (g), it follows that

$$\left[ \frac{\dot{m}\sqrt{T_0}}{P_0} \right]_g = \left[ \frac{\dot{m}\sqrt{T_0}}{P_0} \right]_a \frac{\left\{ \right\}_a}{\left\{ \right\}_g} \quad (A-2)$$

where

$$\left\{ \right\} = \sqrt{\frac{1}{\gamma \cdot mw}} \left[ 1 + \frac{\gamma - 1}{2} M^2 \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

at corresponding operating points.

Similarly, the parameters  $U/a$  may be expressed in terms of the following relation.

$$\frac{U}{a} = \frac{N}{\sqrt{T_0}} \sqrt{\frac{mw}{\gamma}} \left[ 1 + \frac{\gamma - 1}{2} M^2 \right]^{1/2} \quad (A-3)$$

Again, considering the operation of the same compressor with different gases, it follows that

$$\left[ \frac{N}{\sqrt{T_0}} \right]_g = \left[ \frac{N}{\sqrt{T_0}} \right]_a \cdot \frac{\left[ \right]_a}{\left[ \right]_g} \quad (A-4)$$

$$\text{where } \left[ \right] = \sqrt{\frac{mw}{\gamma}} \cdot \left[ 1 + \frac{\gamma - 1}{2} M^2 \right]^{1/2}$$

at corresponding operating points.

It may be pointed out that under the one-dimensional flow approximation corresponding operating points are assumed to yield the same pressure ratio and efficiency. In order to take into account blade passage, blade clearance and inter blade-row spacing, a more sophisticated approach than one-dimensional flow is required.

It is usual practice to express the  $\frac{\dot{m}\sqrt{T_0}}{P_0}$  and  $\frac{N}{\sqrt{T_0}}$  parameters as



corrected quantities by using reference standard values of inlet-pressure and temperature. The corrected mass flow parameter and rotational speed parameter for a compressor are designated as

$$\frac{\dot{m}\sqrt{\theta}}{\delta} \text{ and } \frac{N}{\sqrt{\theta}} ,$$

respectively,

Throughout the analysis, unless it is specified that the performance of a particular stage is being presented with respect to its own stage inlet conditions, all of the compressor performance maps are presented in terms of corrected mass flow and corrected speed.

## APPENDIX B

### PURDUE UNIVERSITY COMPRESSOR PERFORMANCE PROGRAM-PUCPP-001

The purpose of this program is to calculate the stage and overall compressor performance using a one-dimensional technique. The analysis is limited to a six-stage compressor for which exists stage data relating axial velocity to stage pressure rise and stage efficiency. The following four sections describe the necessary input data, the resulting output data, the one-dimensional method and capabilities of the program, and the program listing.

1. Input Data: There are two items of input: (i) stage performance and geometry data, and (ii) inlet and operating conditions. Both integer and real input formats are used. Integer numbers are punched in fields of five locations starting with column 1. No decimal points may be used. Integer fields are right-hand justified. Real numbers are punched in fields of ten locations starting with column 1. Decimal points must be used for real numbers if right-hand justification in each field is to be ignored.

A listing of the input cards is given in the following chart, where one line represents one card.

N	NDATA					
RR	A					} Occurs N times
PHI	SI	ETA	-	Occurs NDATA times		
NFLOW						
FLOW	RPM	T01	P01	CP	W1	- Occurs NFLOW times

Variable Definitions - The input data must have units corresponding to the dimensions given below (where applicable).

Length	feet
Time	seconds
Mass	pound-mass
Force	pound-force
Temperature	Degrees Rankine

The one exception to this is the input value of the compressor rotational speed, revolutions per minute.

N	Number of compressor stages $N \leq 6$ .
NDATA	Number of data points to be input that give stage performance information. There must be NDATA data points for <u>each</u> stage $NDATA \leq 15$ .
RR	Radius at which calculation is to be made. This may be an average radius for the stage. There must be N values of RR input. Units:feet.
A	Stage inlet cross-sectional area. There must be N values of A input. Units:feet <sup>2</sup> .
PHI	Stage performance information - stage inlet axial velocity/rotor speed (determined at RR). There must be NDATA values of PHI input for <u>each</u> stage.
SI	Stage performance information - stage pressure rise/ stage inlet dynamic head. SI must correspond to <u>each</u> value of PHI.
ETA	Stage performance information - stage adiabatic efficiency. ETA must correspond to <u>each</u> value of PHI.

It is necessary to input the stage performance information (PHI,SI and ETA) in ascending order beginning with the smallest value of PHI, for each stage.

NFLOW	Number of mass flows to be input.
FLOW	Operating mass flow. There must be NFLOW values of FLOW input. Each value of FLOW must have the remaining variables associated with it. Units:pound mass/seconds.

RPM	Compressor rotational speed. Must correspond to <u>each</u> value of FLOW. Units:minute <sup>-1</sup>
T01	Compressor inlet total temperature. Must correspond to <u>each</u> value of FLOW. Units:Degrees Rankine.
P01	Compressor inlet total pressure. Must correspond to <u>each</u> value of FLOW. Units:pound-force/feet <sup>2</sup> .
CP	Specific heat at constant pressure. Assumed to be constant throughout compressor. Must correspond to <u>each</u> value of FLOW. Units:feet-pound-force/pound-mass/Deg. Rankine
W1	Compressor inlet molecular weight. Must correspond to <u>each</u> value of FLOW. Units:pound-mass/pound-mole

2. Output Data: There are two types of output data: (i) compressor performance, and (ii) input errors. Providing no input errors have occurred. the program will give values for the following.

1. Compressor Total Pressure Ratio
2. Compressor Total Temperature Ratio
3. Compressor Adiabatic Efficiency
4. Stage Performance Characteristics
  - stage total pressure ratio
  - stage total temperature ratio
  - stage adiabatic efficiency

In addition, the input inlet operating conditions corresponding to these performance values will be printed.

In the event that the input inlet operating conditions cause the stage performance to exceed the limits of information (PHI, SI and ETA) that has been input for that stage, no extrapolation is carried out. Instead, the analysis for that operating point ceases and the program continues with the next operating point.



The following message is printed:

STAGE X DATA IS NOT SUFFICIENT - CALCULATION

TERMINATED FOR Y MASS FLOW,

where X is the stage where the limit was exceeded and Y is the mass flow under consideration. If the compressor performance for that mass flow is desired, the stage performance information for stage X must be expanded.

3. General One-Dimensional Method: The analysis is initiated with the input values of the inlet operating conditions. A value of  $\phi$  is determined which is then compared with the stage performance data input to the program (PHI).

$$\phi_{\text{calc}} = v_a/u = \frac{60\dot{m}}{\rho A 2\pi r R P m} \quad (\text{B-1})$$

upon determining two values of PHI stored in input files that flank the calculated value of  $\phi_{\text{calc}}$ , a weighting factor is determined:

$$\text{Wt. Fact.} = (\phi_2 - \phi_{\text{calc}})/(\phi_2 - \phi_1) \quad (\text{B-2})$$

where  $\phi_2$  = PHI immediately greater than  $\phi_{\text{calc}}$

$\phi_1$  = PHI immediately less than  $\phi_{\text{calc}}$

It is assumed that a straight line connects the points input as stage performance information. Thus, with the above weighting factor it is possible to determine the values of  $\psi$  and  $\eta$  (input as SI and ETA, respectively) corresponding to  $\phi_{\text{calc}}$ . Thus,

$$\psi_{\text{calc}} = (\psi_1 - \psi_2)\text{Wt. Fact.} + \psi_2 \quad (\text{B-3})$$

and

$$\eta_{calc} = (\eta_1 - \eta_2) \text{ Wt. Fact.} + \eta_2, \quad (B-4)$$

where  $(\psi_1 \text{ and } \eta_1)$  and  $(\psi_2 \text{ and } \eta_2)$  correspond to  $(\phi_1)$  and  $(\phi_2)$ , respectively.

The stage pressure rise is found from  $\psi_{calc}$ :

$$\Delta P_o = \psi_{calc} \times (1/2 \rho V_a^2) \quad (B-5)$$

where  $V_a$  is the stage inlet axial velocity. The stage temperature ratio is found by, (i) assuming isentropic flow,

$$T_{o2}/T_{o1} = (P_{o2}/P_{o1})^{(\gamma-1)/\gamma} \quad (B-6)$$

(ii) then accounting for losses by the use of the stage adiabatic efficiency in the following manner:

$$\Delta T_{o_{actual}} = \Delta T_{o_{isentropic}} / \eta_{adiab} \quad (B-7)$$

where,  $\Delta T_{o_{isentropic}}$  is found using equation (6), and (iii)

$$(T_{o2}/T_{o1})_{actual} = (T_{o1} + \Delta T_{o_{actual}}) / T_{o1}. \quad (B-8)$$

The stage and compressor adiabatic efficiency is determined by,

$$\eta_{adiab} = \frac{(P_{o2}/P_{o1})^{(\gamma-1)/\gamma} - 1}{T_{o2}/T_{o1} - 1}, \quad (B-9)$$

where 2 represents the exit, and 1 the inlet, of the stage or compressor, respectively.

```

1.  DIMENSION PHI(6,15), SI(6,15), ETA(6,15), RR(6), A(6)
2.  DIMENSION SPH(6), STR(6), SETA(6)
3.  R=1545.0
4.  G=32.174
5.  PIE=3.1415927
6.  WRITE (6,109)
7.  READ (5,110) N,NDATA
8.  DO 102 I=1,N
9.  READ (5,111) RR(I),A(I)
10. DO 101 K=1,NDATA
11. READ (5,112) PHI(I,K),SI(I,K),ETA(I,K)
12. 101 CONTINUE
13. 102 CONTINUE
14. READ (5,113) NFLOW
15. DO 108 L=1,NFLOW
16. STOR1=1.0
17. STOR2=1.0
18. READ (5,114) RPM,T01,P01,CP,W1,FLOW
19. GAMA=1.0/(1.0-R/(CP*W1))
20. GAMAF=(GAMA-1.0)/GAMA
21. DO 105 I=1,N
22. U=2.0*PIE*RR(I)*RPM/60.0
23. RHU=P01*W1/(K*T01)
24. VA=FLOW/(RHU*A(I))
25. PHII=VA/U
26. K=1
27. IF ((PHII-PHI(I,1)).GE.0.0) GO TO 104
28. 103 WRITE (6,115) I,FLOW
29. GO TO 108
30. 104 K=K+1
31. IF (K.EQ.(NDATA+1)) GO TO 103
32. IF (PHII-PHI(I,K).GT.0.0) GO TO 104
33. FACT=(PHI(I,K)-PHII)/(PHI(I,K)-PHI(I,K-1))
34. SII=-FACT*(SI(I,K)-SI(I,K-1))+SI(I,K)
35. ETAI=-FACT*(ETA(I,K)-ETA(I,K-1))+ETA(I,K)
36. DELP=SI*0.5*RHU*U**2/G
37. P02=P01*DELP
38. SPR(I)=P02/P01
39. TTR=(SPR(I)**GAMAF
40. DELTA=(TTR*T01-T01)/LTAI
41. T02A=T01+DELTA
42. STR(I)=T02A/T01
43. SETA(I)=(SPH(I)**GAMAF-1.0)/(STR(I)-1.0)
44. P01=P02
45. 105 T01=T02A
46. DO 106 I=1,N
47. TPR=SPR(I)*STOR1
48. TTR=STR(I)*STOR2
49. STOR1=TPR
50. 106 STOR2=TTR
51. TETA=(TPR**GAMAF-1.0)/(TTR-1.0)
52. WRITE (6,116) FLOW,RPM,T01,P01,W1
53. WRITE (6,117) TPR,TTR,TETA
54. WRITE (6,118)
55. DO 107 KK=1,N
56. WRITE (6,119) KK,SPR(KK),STR(KK),SETA(KK)
57. 107 CONTINUE
58. 108 CONTINUE
59. STOP

C
60. 109 FORMAT (1H1,42X,51H PUKDUE UNIVERSITY COMPRESSOR PROGRAM - PUCPP
61. 110 FORMAT (2I5)
62. 111 FORMAT (2F10.0)
63. 112 FORMAT (3F10.0)
64. 113 FORMAT (15)
65. 114 FORMAT (6F10.0)
66. 115 FORMAT (////1X,6H STAGE,12,52H DATA IS NOT SUFFICIENT - CALCULATIO
67. 116 FORMAT (////1X,13H MASS FLOW = ,F10.4/2X,6HRPM = ,F10.4/1X,27H INL
68. 117 FORMAT (//1X,29H COMPRESSOR PRESSURE RATIO = ,F10.4/1X,32H COMPRESS
69. 118 FORMAT (//10X,21H STAGE PRESSURE RATIO,10X,24H STAGE TEMPERATURE R
70. 119 FORMAT (/2X,6HSTAGE ,11,6X,F10.6,23X,F10.6,24X,F10.6)

C
71. END

```

## APPENDIX C

### PURDUE UNIVERSITY COMPRESSOR PERFORMANCE PROGRAM-PUCPP-002

During the analytical examination of the N.G.T.E. Reference #109 Compressor it was desired to determine the effects of mass flow bleed and injection toward negating the performance change caused by the liquid phase evaporation. This program was developed to specify the amount of mass flow bleed and/or injection needed to return the compressor to the point which give the same pressure ratio as its original operating point. The analysis also includes the possibility of heating or cooling the injected gas, as desired by the user. The following four sections describe the necessary input data, the resulting output data, the method and capabilities of the program and the program listing.

1. Input Data: There are two items of input: (i) the corrected performance data at the operating point, and the necessary inlet conditions used to obtain their corrected forms, and (ii) factors that govern the injection process. Both integer and real input formats are used. Integer numbers are punched in fields of two locations starting with column 1. No decimal points may be used. Real numbers are punched in fields of ten locations starting with column 1. Decimal points must be used for real numbers if right-hand justification in each field is to be ignored.

A listing of the input cards is given in the following chart, where one line represents one card.



Note that Figure 2.9. of Report No. M-WPAFB-77-1 which is referred to here is included at the end of this Appendix as Figure 6.1.

N

WC1	WCA	RC1	RCA	RPM2			] Occurs N Times
W1	T01	P01	FLOW1	WI	TOI		
CP	DELTP	DELTN	TMAXP	TMAXN	XMAX	YMAX	

Variable definitions - The input data must have units corresponding to the dimensions given below (were applicable).

Length	feet
Time	seconds
Mass	pound-mass
Force	pound-force
Temperature	Degrees Rankine

The one exception involves the units of RPM2, which are per minute.

N	Number of calculations to be made.
WC1	Corrected mass flow of point 1 (see Figure 6.1)
WCA	Corrected mass flow of point a (see Figure 6.1)
RC1	Corrected rotor speed of point 1 (see Figure 6.1)
RCA	Corrected rotor speed of point a (see Figure 6.1)
RPM2	Compressor speed at point 2. This may be arbitrarily chosen. Units:minute <sup>-1</sup> .
W1	Molecular weight of conditions at point 1 (see Figure 6.1) Units:pound-mass/pound-mole.
T01	Total temperature of conditions at point 1 (see Figure 6.1)Units:Deg. Rankine.
P01	Total pressure of conditions at point 1 (see Figure 6.1)Units:pound-force/feet <sup>2</sup> .
FLOW1	Mass flow at point 1 (see Figure 6.1) Units:pound-mass/second

- WI Molecular weight of gas to be injected.  
Units:pound-mass/pound-mole.
- TOI Total temperature of gas to be injected, before heating or cooling occurs. Units:Degrees Rankine.
- CP Specific heat at constant pressure. Assumed to be constant throughout compressor.  
Units:feet-pounds-force/pound-mass/Degrees Rankine
- DELTP Specifies the incremental change in temperature of heated injected air that is desired during the analysis. For a given mass flow, increments of DELTP will be added to the injection temperature and analyzed until TMAXP is exceeded.  
Units:Degrees Rankine.
- TMAXP Maximum temperature for heated injected gas. Note that TMAXP/DELTP must be an integer.  
Units:Degrees Rankine
- DELTN Specifies the incremental change in temperature of the cooled injected gas that is desired during the analysis. For a given mass flow, increments of DELTN will be subtracted from the injection temperature and analyzed until TMAXN is exceeded.
- TMAXN Minimum temperature for cooled injected gas. Note that TMAXN/DELTN must be an integer.  
Units:Degrees Rankine.
- XMAX Maximum injection mass flow ratio. This eliminates physically unrealizable bleed rates.
- YMAX Maximum bleed mass flow ratio. This eliminates physically unrealizable bleed rates.

Note that  $TMAXP/DELTP + TMAXN/DELTN \leq 20$ .

To insure correct non-dimensional variables being input, the following equation should be used.

$$(WC)_x = \left( \frac{\dot{m} \sqrt{T_{01}}}{P_{01}} \right)_x \quad C-1$$

$$(RC)_x = \left( \frac{N}{\sqrt{T_{01}}} \right)_x \quad C-2$$

where,  $\dot{m}$  = mass flow

$N$  = rotor rotational speed

$T_{01}$  = stage inlet total temperature

$P_{01}$  = stage inlet total pressure

$WC$  = corrected mass flow

$RC$  = corrected RPM

$( )_x$  = pertaining to the operating point  $x$  ( $x = a, 1$ )

2. Output Data: There are two types of output data: (i) the injection and bleed mass flows for the given operating conditions, and (ii) diagnostic explanation concerning physically impossible developments.

Providing the latter does not occur, the program will give the following results:

1. Injection mass flow ratio,  $x$

2. Bleed mass flow ratio,  $y$ .

In addition, the input data and injection conditions corresponding to these input data will be printed. If values of  $x$  and/or  $y$  are negative or greater than the maximum limits, as given by  $XMAX$  and  $YMAX$  respectively, the following statements will be printed:

VALUES OF  $X$  OR  $Y$  ARE NEGATIVE

VALUES OF  $X$  OR  $Y$  EXCEED LIMIT

respectively.

3. General Equations and Method: Figure 6.1 illustrates the desired result of returning a certain operating point to the position which give the same pressure ratio as its original position by changing the corrected parameters associated with the point. The method of altering

the parameters is the bleeding and injection at particular stage inlet. If point (1) (in Fig. 6.1) is to return to the point (2) which give the same pressure ratio as its original point (a), it is necessary that the following expressions be satisfied.

$$(SP)_a = (SP)_2 \quad C-3$$

$$(MP)_a = (MP)_2 \quad C-4$$

where the subscript a and 2 corresponds to the point (a) and (2) in Figure 6.1, respectively, and this will hold throughout this analysis and where

$$SP = \text{Speed Parameter} \equiv \frac{N}{\sqrt{T_0}} \sqrt{\frac{mw}{\gamma}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/2}$$

$$MP = \text{Mass Flow Parameter} \equiv \frac{\dot{m} \sqrt{T_0}}{P_0} \sqrt{\frac{1}{\gamma \cdot mw}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

$T_0$  = Total Temperature

$P_0$  = Total Pressure

$mw$  = molecular weight

$\gamma$  = specific heat ratio

$M$  = Mach number



That is,

$$Nc_a \sqrt{\frac{mw_a}{\gamma_a}} \left(1 + \frac{\gamma_a - 1}{2} M_a^2\right)^{\frac{1}{2}} = \frac{N_2}{\sqrt{T_{02}}} \sqrt{\frac{mw_2}{\gamma_2}} \left(1 + \frac{\gamma_2 - 1}{2} M_2^2\right)^{\frac{1}{2}} \quad C-5$$

$$Mc_a \sqrt{\frac{1}{\gamma_a \cdot mw_a}} \left(1 + \frac{\gamma_a - 1}{2} M_a^2\right)^{\frac{\gamma_a + 1}{2(\gamma_a - 1)}}$$

$$= \frac{\dot{m}_2 \sqrt{T_{02}}}{P_{02}} \sqrt{\frac{1}{\gamma_2 \cdot mw_2}} \left(1 + \frac{\gamma_2 - 1}{2} M_2^2\right)^{\frac{\gamma_2 + 1}{2(\gamma_2 - 1)}} \quad C-6$$

where  $Nc_a = \left(\frac{N}{\sqrt{T_0}}\right)_a$  and  $Mc_a = \left(\frac{\dot{m} \sqrt{T_0}}{P_0}\right)_a$

The values of  $Nc_a$  and  $Mc_a$  should be specified by input data. The equations ( C-5 ) and ( C-6 ) can be written as

$$A = \frac{mw_2}{T_{02} \gamma_2} \quad C-7$$

$$B = (1 + x-y)^2 \frac{T_{02}}{\gamma_2 \cdot mw_2} \quad C-8$$

where

$$A = \left(\frac{Nc_a}{N_2} \sqrt{\frac{mw_a}{\gamma_a}}\right)^2 \cdot \xi \quad C-9$$

$$B = \left( \frac{M c_a P_1}{\dot{m}_1} \sqrt{\frac{1}{\gamma_a \cdot m w_a}} \right)^2 \cdot \eta \quad C-10$$

$$\xi = \frac{1 + \frac{\gamma_a - 1}{2} M_a^2}{1 + \frac{\gamma_2 - 1}{2} M_2^2} \quad C-11$$

$$\eta = \frac{\left( 1 + \frac{\gamma_a - 1}{2} M_a^2 \right) \frac{\gamma_a + 1}{\gamma_a - 1}}{\left( 1 + \frac{\gamma_2 - 1}{2} M_2^2 \right) \frac{\gamma_2 + 1}{\gamma_2 - 1}} \quad C-12$$

In deriving the equations ( C-7 ) and ( C-8 ), it was assumed that the pressure is unaffected by small amounts of injection and/or bleed and the following relationship was used.

$$\dot{m}_2 = \dot{m}_1 (1 + x - y) \quad C-13$$

where

$$x = \frac{\dot{m}_{\text{inject}}}{\dot{m}_1}$$

$$y = \frac{\dot{m}_{\text{bleed}}}{\dot{m}_1}$$

The molecular weight of mixture after injection and/or bleed,  $mw_2$ , can be determined as follows.

$$mw_2 = mw_1 \frac{\dot{m}_1}{\dot{m}_2} + mw_i \frac{\dot{m}_i}{\dot{m}_2} - mw_b \frac{\dot{m}_b}{\dot{m}_2} \quad C-14$$

where

$\dot{m}_1, mw_1$  = mass flow and molecular weight of mixture corresponding to point (1)

$\dot{m}_2, mw_2$  = mass flow and molecular weight of mixture corresponding to point (2)

$\dot{m}_i, mw_i$  = mass flow and molecular weight of mixture to be injected

$\dot{m}_b, mw_b$  = mass flow and molecular weight of mixture to bleed

Assuming  $mw_b = mw_1$ , Eq. (A.3.14) becomes

$$mw_2 = \frac{(1 - y) \cdot mw_1 + x \cdot mw_i}{1 + x - y} \quad C-15$$

In the same manner, the total temperature corresponding to point (2),  $T_2$ , is determined by

$$T_{02} = \frac{(1 - y) T_{01} + z \cdot x \cdot T_{0i}}{1 + x - y} \quad C-16$$

where

$T_{0i}$  = total temperature of mixture to be injected

$T_{01}$  = total temperature of mixture corresponding to point (1)

$z$  = multiplicative factor relating the heated or cooled injection total temperature to  $T_i$   
 $(T_{\text{heated}}/T_i \text{ or } T_{\text{cooled}}/T_i)$

Assuming that the specific heat at constant pressure is constant throughout compressor, the specific heat ratio corresponding to point (2),  $\gamma_2$ , is given by

$$\gamma_2 = \frac{1}{1 - \frac{Ru}{C_p \cdot mw_2}} \quad C-17$$

where  $C_p$  = specific heat at constant pressure

$Ru$  = universal gas constant

Substituting equations ( C-15 ), ( C-16 ), and ( C-17 ) into equation ( C-7 ) and ( C-8 ), and solving for  $x$  and  $y$ , one can obtain

$$x = \frac{E}{F} \frac{(mw_i + C \cdot mw_1)}{1 + C} \quad C-18$$

$$y = 1 - C x \quad C-19$$

where

$$C = \frac{zT_{0i}A - mw_i + Ru/C_p}{-T_{01}A + mw_1 - Ru/C_p} \quad C-20$$

$$A = \left( \frac{Nc_a}{N_2} \sqrt{\frac{mw_a}{\gamma_a}} \right)^2 \cdot \xi \quad C-21$$

$$E = \frac{Mc_a P_1}{m_1} \sqrt{\frac{1}{\gamma_a \cdot mw_a}} \cdot \sqrt{\eta} \quad C-22$$



$$D = (zT_i + CT_1) [mw_i + C.mw_1 - \frac{Ru}{C_p} (1 + C)] \quad C-23$$

$$F = \sqrt{D} \quad C-24$$

Equations ( C-18 ) through ( C-24 ) are utilized in the program to directly determine values of x and y. It is also of interest to determine the values of x and y for the cases of injection only and bleed only, respectively. At present, only the case of bleed only has been incorporated into the program. For this condition, the following equations must be satisfied.

$$N_2 = \frac{Nc_a \sqrt{\frac{mw_a}{\gamma_a}} \sqrt{T_{01}} \sqrt{\xi}}{\sqrt{mw_1 - \frac{Ru}{C_p}}} \quad C-25$$

$$y = 1 - \frac{E}{\sqrt{\frac{T_{01}}{mw_1}} (1 - \frac{Ru}{C_p \cdot mw_1})} \quad C-26$$

In the present case, the value of  $\xi$  and  $\eta$ , which are given by Eqs. (C-11 and C-12 ), are considered to be nearly one. Thus, assuming that  $\xi = \eta = 1$ , the program was developed to specify the amount of mass flow bleed and/or injection needed to return the compressor to the point which give the same pressure ratio as its original operating point.

#### 4. The Program Listing:

```

1.      DIMENSION Z(21), X(21), Y(21)
2.      INTEGER TINCPC
3.      INTEGER TINCNC
4.      H=1545.0
5.      WRITE (6,108)
6.      108 FORMAT (1H1.40X,51H PURDUE UNIVERSITY COMPRESSOR PROGRAM - PUCPP
7.      1-002////////)
8.      READ (5,109) N
9.      109 FORMAT (I2)
10.     DO 107 I=1,N
11.     READ (5,110) WC1,WCA,RC1,RCA,RPM2
12.     110 FORMAT (5F10.0)
13.     READ (5,111) W1,T01,P01,FLOW1,WI,T0I
14.     111 FORMAT (6F10.0)
15.     READ (5,112) CP,DFLTP,DELTP,TMAXP,TMAXN,XMAX,YMAX
16.     112 FORMAT (7F10.0)
17.     WRITE (6,113) WC1,WCA,RC1,RCA,RPM2,W1,T01,P01,FLOW1,WI,T0I,CP,
18.     1DELTP,DELTP,TMAXP,TMAXN,XMAX,YMAX
19.     113 FORMAT (////1X,'INPUT DATA'/1X,'WC1=' ,F10.4,5X,'WCA=' ,F10.4,5X,
20.     2'RC1=' ,F10.4,5X,'RCA=' ,F10.4,5X,'RPM2=' ,F10.4,5X/1X,'W1=' ,F10.4,
21.     35X,'T01=' ,F10.4,5X,'P01=' ,F10.4,5X,'FLOW1=' ,F9.4,5X/1X,'WI=' ,F9.6,
22.     45X,'T0I=' ,F10.4,5X,'CP=' ,F10.4,5X,'DELTP=' ,
23.     5F10.4,5X,'DELTP=' ,F10.4,5X/1X,'TMAXP=' ,F10.4,5X,'TMAXN=' ,F10.4,5X,
24.     6'XMAX=' ,F10.4,5X,'YMAX=' ,F10.4,5X//)
25.     BNEG=1.0
26.     Z(1)=1.0
27.     IF (ABS(DELTP).LT..0001) DELTP=1.0
28.     IF (ABS(DELTPN).LT..0001) DELTPN=1.0
29.     TINCPC=TMAXP/DELTP+1.0
30.     TINCNC=TMAXN/DELTPN
31.     A=(RCA/RPM2)**2*(28.964/1.40)
32.     E=(WCA*P01/FLOW1)/SQRT(1.40)/SQRT(28.964)
33.     KK=TINCPC+TINCNC
34.     DELT=DELTP
35.     DO 102 K=1,KK
36.     IF (K.EQ.1) GO TO 101
37.     Z(K)=(T0I+BNEG*DELTP)/T0I
38.     DELT=DELTP
39.     101 C=(Z(K)*A*T0I-WI+R/CP)/(W1-A*T0I-R/CP)
40.     D=(Z(K)*T0I+C*T01)*(WI+C*W1-R/CP*(1.0+C))
41.     F=D**.5
42.     X(K)=E/F*(WI+C*W1)/(1.0+C)
43.     Y(K)=1.0-C*X(K)
44.     IF (K.NE.TINCPC) GO TO 102
45.     BNEG=-1.0
46.     DELTP=DELTPN
47.     DELT=DELTPN
48.     102 CONTINUE
49.     RPM22=RCA/(W1/T01-R/(CP*T01))**.5*SQRT(28.964)/SQRT(1.4)
50.     YY=1.0-E/(T01/W1-R/CP*T01/(W1)**2)**.5
51.     T022=T01
52.     FLOW22=FLOW1*(1.0-YY)
53.     P02=P01
54.     RC22=RPM22/SQRT(T022)
55.     WC22=FLOW22/P02*SQRT(T022)
56.     IF (ABS(RPM2-RPM22)/RPM2).LE..001) GO TO 106
57.     DO 105 L=1,KK
58.     T0=Z(L)*T0I
59.     IF (X(L).GE.0.0.AND.Y(L).GE.0.0) GO TO 103
60.     WRITE (6,115) T0
61.     115 FORMAT (//1X,'T0ADN=' ,F10.4,10X,30H VALUES OF X OR Y ARE NEGATIVE)
62.     GO TO 105
63.     103 IF (X(L).LE.XMAX.AND.Y(L).LE.YMAX) GO TO 104
64.     WRITE (6,116) T0
65.     116 FORMAT (//1X,'T0A00=' ,F10.4,10X,'VALUES OF X OR Y EXCEED LIMIT ')
66.     GO TO 105
67.     104 T02=((1.0-Y(L))*T01+T0*X(L))/(1.0*X(L)-Y(L))
68.     FLOW2=FLOW1*(1.0+X(L)-Y(L))
69.     P02=P01
70.     RC2=RPM2/SQRT(T02)
71.     WC2=FLOW2/P02*SQRT(T02)
72.     WRITE (6,117) T0,X(L),Y(L),T02,FLOW2,P02,RC2,WC2
73.     117 FORMAT (//1X,'T0ADN=' ,F9.4,3X,'X=' ,F7.4,3X,'Y=' ,F7.4,3X,'T02=' ,F9.4,
74.     1.3X,'FLOW2=' ,F9.4,3X,'P02=' ,F9.4,3X,'RC2=' ,F9.4,3X,'WC2=' ,F9.4)
75.     105 CONTINUE
76.     IF (YY.LE.YMAX.AND.YY.GE.0.0) GO TO 121
77.     WRITE (6,120)
78.     120 FORMAT (//1X,'THIS CANNOT BE ACCOMPLISHED WITH BLEED ONLY')
79.     GO TO 107
80.     121 WRITE (6,118) RPM22,YY,RC22,WC22
81.     118 FORMAT (//1X,'FOR BLEED ONLY',5X,'RPM=' ,F10.4,5X,'Y=' ,F7.4,5X,
82.     2'RC2=' ,F10.4,5X,'WC2=' ,F10.5)
83.     GO TO 107
84.     106 WRITE (6,119) RPM22,YY,RC22,WC22
85.     119 FORMAT (//1X,'THIS MAY BE ACCOMPLISHED WITH BLEED ONLY',5X,'RPM=' ,
86.     3F10.4,5X,'Y=' ,F7.4,5X,'RC2=' ,F10.4,5X,'WC2=' ,F10.5)
87.     107 CONTINUE
88.     STOP
89.
90. C
91. C
92. END

```

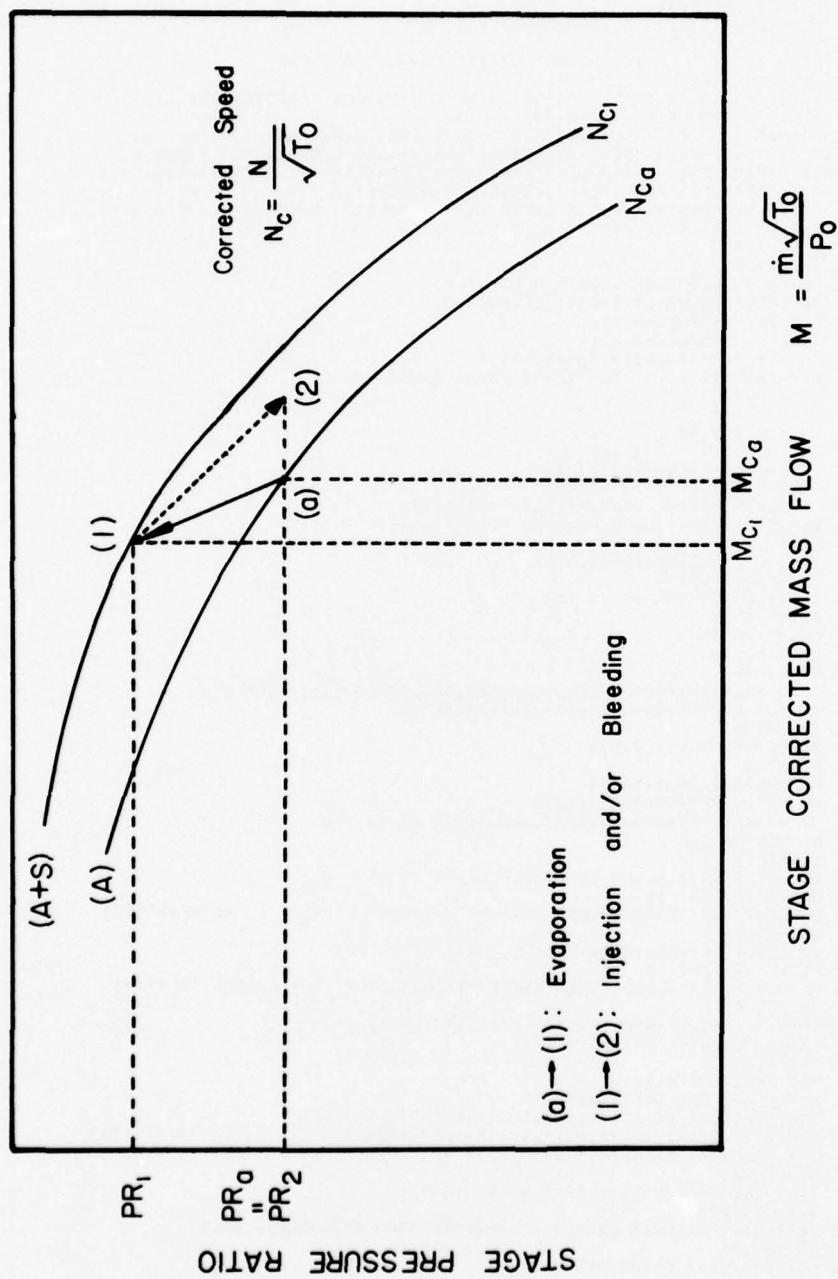


Figure 6.1 Illustration for Change of Compressor Operating Condition

## Appendix D

### EQUATIONS OF MOTION FOR TWO PHASE FLOW

#### Nomenclature

Reference should be made to Report No. AFAPL-TR-76-77 for the nomenclature except as noted below.

- $E_{I_i}$  internal energy per unit mass of the class  $i$  droplets ( $i=1,2$ )
- $E_\psi$  energy required in connection with forces stem from momentum change because of interaction between the two classes of droplets
- $E_s$  energy required for the change of droplet surface area during formation of new droplets
- $\dot{m}_c$  mass that would be created due to condensation per unit volume of mixture
- $\dot{m}_e$  mass that would be lost due to evaporation per unit volume of mixture
- $\bar{m}_1$  average mass of a droplet of class I
- $N_1$  total number of droplets generated per unit volume per unit time due to break up
- $\alpha_D$  bulk volume concentration of drizzle in the mixture
- $\psi_i$  forces exerted on the phase of class  $i$  droplets by the other class of droplets ( $i=1,2$ )



# I. Three-Dimensional Conservation Equations in Intrinsic Coordinates

## (a) Mass conservation Equations.

### Gas Phase:

$$\begin{aligned} \frac{1}{r} \left[ \frac{\partial}{\partial m} (r W_{gm} [(1-\sigma_v - \alpha_b) \rho_g + \rho_b]) + \frac{\partial}{\partial \theta} (W_{g\theta} [(1-\sigma_v - \alpha_b) \rho_g \right. \\ \left. + \rho_b]) + \frac{\partial}{\partial n} (r W_{gn} [(1-\sigma_v - \alpha_b) \rho_g + \rho_b]) + \frac{\partial}{\partial m} (r U_m [(1-\sigma_v - \alpha_b) \rho_g \right. \\ \left. + \rho_b]) + \frac{\partial}{\partial \theta} (U_\theta [(1-\sigma_v - \alpha_b) \rho_g + \rho_b]) + \frac{\partial}{\partial n} (r U_n [(1-\sigma_v - \alpha_b) \rho_g \right. \\ \left. + \rho_b]) \right] = (\dot{m}_{e_1} - \dot{m}_{c_1}) + (\dot{m}_{e_2} - \dot{m}_{c_2}) \end{aligned} \quad (D-1)$$

### Liquid Phase:

#### 1. Droplet - Type 1:

$$\begin{aligned} \frac{1}{r} \left[ \frac{\partial}{\partial m} (r \rho W_{pm1}) + \frac{\partial}{\partial \theta} (\rho W_{p\theta1}) + \frac{\partial}{\partial n} (r \rho W_{pn1}) + \frac{\partial}{\partial m} (r \rho U_m) + \frac{\partial}{\partial \theta} (\rho U_\theta) \right. \\ \left. + \frac{\partial}{\partial n} (r \rho U_n) \right] = (\dot{m}_{c_1} - \dot{m}_{e_1}) - M_1 + N_1 \bar{m}_1 \end{aligned} \quad (D-2)$$

#### 2. Droplet - Type 2:

$$\begin{aligned} \frac{1}{r} \left[ \frac{\partial}{\partial m} (r \rho_2 W_{pm2}) + \frac{\partial}{\partial \theta} (\rho_2 W_{p\theta2}) + \frac{\partial}{\partial n} (r \rho_2 W_{pn2}) + \frac{\partial}{\partial m} (r \rho_2 U_m) \right. \\ \left. + \frac{\partial}{\partial \theta} (\rho_2 U_\theta) + \frac{\partial}{\partial n} (r \rho_2 U_n) \right] \\ = (\dot{m}_{c_2} - \dot{m}_{e_2}) + M_1 - N_1 \bar{m}_1 \end{aligned} \quad (D-3)$$

(b) Momentum Conservation Equations.

Gas Phase:

m-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \right] \left[ (W_{gm} + U_m) \left( \frac{\partial W_{gm}}{\partial m} + \frac{\partial U_m}{\partial m} \right) \right. \\ & \left. - \frac{1}{r} (W_{g\theta} + U_\theta) \left( (W_{g\theta} + U_\theta) \frac{\partial r}{\partial m} - \frac{\partial W_{gm}}{\partial \theta} - \frac{\partial U_m}{\partial \theta} \right) \right] \\ & = - \frac{\partial P_g}{\partial m} + F_{Bgm} + F_{Vgm} + F_{Im} \end{aligned} \quad (D-4)$$

$\theta$ -component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \right] \left[ \frac{1}{r} (W_{gm} + U_m) \left( \frac{\partial r W_{g\theta}}{\partial m} + \frac{\partial r U_\theta}{\partial m} \right) \right. \\ & \left. + \frac{1}{r} (W_{g\theta} + U_\theta) \left( \frac{\partial W_{g\theta}}{\partial \theta} + \frac{\partial U_\theta}{\partial \theta} \right) \right] = - \frac{1}{r} \frac{\partial P_g}{\partial \theta} + F_{Bg\theta} + F_{Vg\theta} + F_{I\theta} \end{aligned} \quad (D-5)$$

n-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \right] \left[ (W_{gn} + U_n) \left( \frac{\partial W_{gn}}{\partial m} + \frac{\partial U_n}{\partial m} \right) \right. \\ & \left. - \frac{1}{r} (W_{g\theta} + U_\theta) \left( (W_{g\theta} + U_\theta) \frac{\partial r}{\partial n} - \frac{\partial W_{gn}}{\partial \theta} - \frac{\partial U_n}{\partial \theta} \right) \right] \\ & = - \frac{\partial P_g}{\partial n} + F_{Bgn} + F_{Vgn} + F_{In} \end{aligned} \quad (D-6)$$

Liquid Phase:

1. Droplet - Type 1:

m-component

$$\begin{aligned} \rho_i \left[ (W_{pm1} + U_m) \left( \frac{\partial W_{pm1}}{\partial m} + \frac{\partial U_m}{\partial m} \right) - \frac{1}{r} (W_{p\theta1} + U_\theta) \cdot \right. \\ \left. \left( (W_{p\theta1} + U_\theta) \frac{\partial r}{\partial m} - \frac{\partial W_{pm1}}{\partial \theta} - \frac{\partial U_m}{\partial \theta} \right) \right] = F_{Bpm1} + F_{Vpm1} \\ - F_{Im1} + \psi_{m1} + N_i \bar{m}_i (W_{pm2} + U_m) \\ - M_i (W_{pm1} + U_m) \end{aligned} \quad (D-7)$$

\theta-component

$$\begin{aligned} \rho_i \left[ \frac{1}{r} (W_{pm1} + U_m) \left( \frac{\partial r W_{p\theta1}}{\partial m} + \frac{\partial r U_\theta}{\partial m} \right) + \frac{1}{r} (W_{p\theta1} + U_\theta) \cdot \right. \\ \left. \left( \frac{\partial W_{p\theta1}}{\partial \theta} + \frac{\partial U_\theta}{\partial \theta} \right) \right] = F_{Bp\theta1} + F_{Vp\theta1} - F_{Ip\theta1} + \psi_{\theta1} \\ + N_i \bar{m}_i (W_{p\theta2} + U_\theta) - M_i (W_{p\theta1} + U_\theta) \end{aligned} \quad (D-8)$$

n-component

$$\begin{aligned} \rho_i \left[ (W_{pn1} + U_n) \left( \frac{\partial W_{pn1}}{\partial n} + \frac{\partial U_n}{\partial n} \right) - \frac{1}{r} (W_{p\theta1} + U_\theta) \left( (W_{p\theta1} + U_\theta) \cdot \right. \right. \\ \left. \left. \frac{\partial r}{\partial n} - \frac{\partial W_{pn1}}{\partial n} - \frac{\partial U_n}{\partial \theta} \right) \right] = F_{Bpn1} + F_{Vpn1} - F_{In1} + \psi_{n1} \\ + N_i \bar{m}_i (W_{pn2} + U_n) - M_i (W_{pn1} + U_n) \end{aligned} \quad (D-9)$$

## 2. Droplet - Type 2:

m-component

$$\begin{aligned} \rho_2 \left[ (W_{pm2} + U_m) \left( \frac{\partial W_{pm2}}{\partial m} + \frac{\partial U_m}{\partial m} \right) - \frac{1}{r} (W_{p\theta2} + U_\theta) \left( (W_{p\theta2} + U_\theta) \cdot \right. \right. \\ \left. \left. \frac{\partial r}{\partial m} - \frac{\partial W_{pm2}}{\partial \theta} - \frac{\partial U_m}{\partial \theta} \right) \right] = F_{Bpm2} + F_{Vpm2} - F_{Im2} + \psi_{m2} \\ + M_1 (W_{pm1} + U_m) - N_1 \bar{m}_1 (W_{pm2} + U_m) \end{aligned} \quad (D-10)$$

$\theta$ -component

$$\begin{aligned} \rho_2 \left[ \frac{1}{r} (W_{pm2} + U_m) \left( \frac{\partial r W_{p\theta2}}{\partial m} + \frac{\partial r U_\theta}{\partial m} \right) + \frac{1}{r} (W_{p\theta2} + U_\theta) \cdot \right. \\ \left. \left( \frac{\partial W_{p\theta2}}{\partial \theta} + \frac{\partial U_\theta}{\partial \theta} \right) \right] = F_{Bp\theta2} + F_{Vp\theta2} - F_{Ip\theta2} + \psi_{\theta2} + M_1 (W_{p\theta1} \\ + U_\theta) - N_1 \bar{m}_1 (W_{p\theta2} + U_\theta) \end{aligned} \quad (D-11)$$

n-component

$$\begin{aligned} \rho_2 \left[ (W_{pn2} + U_n) \left( \frac{\partial W_{pn2}}{\partial m} + \frac{\partial U_n}{\partial m} \right) - \frac{1}{r} (W_{p\theta2} + U_\theta) \left( (W_{p\theta2} + U_\theta) \cdot \right. \right. \\ \left. \left. \frac{\partial r}{\partial n} - \frac{\partial W_{pn2}}{\partial \theta} - \frac{\partial U_n}{\partial \theta} \right) \right] = F_{Bpn2} + F_{Vpn2} - F_{In2} + \psi_{n2} + M_1 \cdot \\ (W_{pn1} + U_n) - N_1 \bar{m}_1 (W_{pn2} + U_n) \end{aligned} \quad (D-12)$$

### (c) Energy Conservation Equations.

Gas Phase:

$$\begin{aligned} [(1 - \sigma_v - \alpha_v) \rho_g + \rho_d] \left[ (W_{gm} + U_m) \left( \frac{\partial h_g}{\partial m} + \frac{\partial h_d}{\partial m} \right) + \frac{1}{r} (W_{g\theta} + U_\theta) \cdot \right. \\ \left. \left( \frac{\partial h_g}{\partial \theta} + \frac{\partial h_d}{\partial \theta} \right) \right] = \Phi_g + \Phi_I + P_g + (W_{gm} + U_m) \frac{\partial P_g}{\partial m} + \frac{1}{r} (W_{g\theta} \\ + U_\theta) \frac{\partial P_g}{\partial \theta} - \frac{\partial \mathcal{E}'_{gm}}{\partial m} - \frac{\partial \mathcal{E}'_{Im}}{\partial m} - \frac{1}{r} \left( \frac{\partial \mathcal{E}'_{g\theta}}{\partial \theta} + \frac{\partial \mathcal{E}'_{I\theta}}{\partial \theta} \right) \\ - \frac{\partial \mathcal{E}'_{gn}}{\partial n} - \frac{\partial \mathcal{E}'_{In}}{\partial n} \end{aligned} \quad (D-13)$$



Liquid Phase:

1. Droplet - Type 1:

$$\begin{aligned} \rho_l \left[ (W_{pm1} + U_m) \frac{\partial u_{pl}}{\partial m} + \frac{1}{r} (W_{p\theta1} + U_\theta) \frac{\partial u_{pl}}{\partial \theta} \right] &= \Phi_{p1} - \Phi_{I1} + \bar{P}_{p1} \\ &- \frac{\partial \mathcal{G}'_{pm1}}{\partial m} - \frac{\partial \mathcal{G}'_{Im1}}{\partial m} - \frac{1}{r} \left( \frac{\partial \mathcal{G}'_{p\theta1}}{\partial \theta} - \frac{\partial \mathcal{G}'_{I\theta1}}{\partial \theta} \right) - \frac{\partial \mathcal{G}'_{pn1}}{\partial n} \\ &+ \frac{\partial \mathcal{G}'_{In1}}{\partial n} - M_1 E_{I1} + M_1 \bar{m}_1 E_{I2} + E_{s1} - E_{s2} - E_{\psi1} \end{aligned} \quad (D-14)$$

2. Droplet - Type 2:

$$\begin{aligned} \rho_l \left[ (W_{pm2} + U_m) \frac{\partial u_{p2}}{\partial m} + \frac{1}{r} (W_{p\theta2} + U_\theta) \frac{\partial u_{p2}}{\partial \theta} \right] &= \Phi_{p2} - \Phi_{I2} + \bar{P}_{p2} \\ &- \frac{\partial \mathcal{G}'_{pm2}}{\partial m} + \frac{\partial \mathcal{G}'_{Im2}}{\partial m} - \frac{1}{r} \left( \frac{\partial \mathcal{G}'_{p\theta2}}{\partial \theta} - \frac{\partial \mathcal{G}'_{I\theta2}}{\partial \theta} \right) - \frac{\partial \mathcal{G}'_{pn2}}{\partial n} \\ &+ \frac{\partial \mathcal{G}'_{In2}}{\partial n} + M_1 E_{I1} - M_1 \bar{m}_1 E_{I2} - E_{s1} + E_{s2} - E_{\psi2} \end{aligned} \quad (D-15)$$

II. Axisymmetric Conservation Equations in Intrinsic Coordinates

(a) Mass Conservation Equations.

Gas Phase:

$$\begin{aligned} \frac{1}{r} \left[ \frac{\partial}{\partial m} \left( r \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] W_{gm} \right) + r \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \frac{\partial W_{gn}}{\partial n} \right] \\ = (\dot{m}_{e1} - \dot{m}_{c1}) + (\dot{m}_{e2} - \dot{m}_{c2}) \end{aligned} \quad (D-16)$$

Liquid Phase:

1. Droplet - Type 1:

$$\frac{1}{r} \left[ \frac{\partial}{\partial m} (r \rho W_{pm1}) + r \rho \frac{\partial W_{pm1}}{\partial n} \right] = (\dot{m}_{c1} - \dot{m}_{e1}) - M_1 + N_1 \bar{m}_1 \quad (D-17)$$

2. Droplet - Type 2:

$$\frac{1}{r} \left[ \frac{\partial}{\partial m} (r \rho W_{pm2}) + r \rho \frac{\partial W_{pm2}}{\partial n} \right] = (\dot{m}_{c2} - \dot{m}_{e2}) + M_1 - N_1 \bar{m}_1 \quad (D-18)$$

(b) Momentum Conservation Equations.

Gas Phase:

m-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gm} \frac{\partial W_{gm}}{\partial m} - \frac{1}{r} (W_{g\theta} + r \omega)^2 \frac{\partial r}{\partial m} \right] \\ & = - \frac{\partial p}{\partial m} + F_{Bgm} + F_{Vgm} + F_{Im} \end{aligned} \quad (D-19)$$

\theta-component

$$\left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \frac{W_{gm}}{r} \left( \frac{\partial r W_{g\theta}}{\partial m} + 2 \omega r \frac{\partial r}{\partial m} \right) = F_{Bg\theta} + F_{Vg\theta} + F_{Ig\theta} \quad (D-20)$$

n-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gn} \frac{\partial W_{gn}}{\partial m} - \frac{1}{r} (W_{g\theta} + r \omega)^2 \frac{\partial r}{\partial n} \right] \\ & = - \frac{\partial p}{\partial n} + F_{Bgn} + F_{Vgn} + F_{In} \end{aligned} \quad (D-21)$$

Liquid Phase:

1. Droplet - Type 1:

$$\begin{aligned} \rho_1 \left[ W_{pm1} \frac{\partial W_{pm1}}{\partial m} - \frac{1}{r} (W_{p\theta1} + r\omega)^2 \frac{\partial r}{\partial m} \right] &= F_{Bpm1} + F_{Vpm1} - F_{Im1} \\ &+ \Psi_{m1} + N_1 \bar{m}_1 (W_{pm2} + U_m) - M_1 (W_{pm1} + U_m) \end{aligned} \quad (D-22)$$

$\theta$ -component

$$\begin{aligned} \rho_1 \frac{W_{pm1}}{r} \left[ \frac{\partial}{\partial m} (r W_{p\theta1}) + 2\omega r \frac{\partial r}{\partial m} \right] &= F_{Bp\theta1} + F_{Vp\theta1} - F_{I\theta1} + \Psi_{\theta1} \\ &+ M_1 \bar{m}_1 (W_{p\theta2} + U_\theta) - M_1 (W_{p\theta1} + U_\theta) \end{aligned} \quad (D-23)$$

$n$ -component

$$\begin{aligned} \rho_1 \left[ W_{pn1} \frac{\partial W_{pn1}}{\partial n} - \frac{1}{r} (W_{p\theta1} + r\omega)^2 \frac{\partial r}{\partial n} \right] &= F_{Bpn1} + F_{Vpn1} - F_{In1} \\ &+ \Psi_{n1} + N_1 \bar{m}_1 (W_{pn2} + U_n) - M_1 (W_{pn1} + U_n) \end{aligned} \quad (D-24)$$

2. Droplet - Type 2:

$m$ -component

$$\begin{aligned} \rho_2 \left[ W_{pm2} \frac{\partial W_{pm2}}{\partial m} - \frac{1}{r} (W_{p\theta2} + r\omega)^2 \frac{\partial r}{\partial m} \right] &= F_{Bpm2} + F_{Vpm2} - F_{Im2} \\ &+ \Psi_{m2} + M_1 (W_{pm1} + U_m) - N_1 \bar{m}_1 (W_{pm2} + U_m) \end{aligned} \quad (D-25)$$

$\theta$ -component

$$\begin{aligned} \rho_2 \frac{W_{pm2}}{r} \left[ \frac{\partial}{\partial m} (r W_{p\theta2}) + 2\omega r \frac{\partial r}{\partial m} \right] &= F_{Bp\theta2} + F_{Vp\theta2} - F_{I\theta2} \\ &+ \Psi_{\theta2} + M_1 (W_{p\theta1} + U_\theta) - N_1 \bar{m}_1 (W_{p\theta2} + U_\theta) \end{aligned} \quad (D-26)$$

n-component

$$\rho_2 \left[ W_{p_{n2}} \frac{\partial W_{p_{n2}}}{\partial m} - \frac{1}{r} (W_{p_{o2}} + r\omega)^2 \frac{\partial r}{\partial n} \right] = \overline{F}_{B_{p_{n2}}} + \overline{F}_{V_{p_{n2}}} - \overline{F}_{I_{n2}} \\ + \Psi_{n2} + M_1 (W_{p_{n1}} + U_n) - N_1 \overline{m}_1 (W_{p_{n2}} + U_n) \quad (D-27)$$

(c) Energy Conservation Equations.

Gas Phase:

$$\left[ (1 - \sigma - \alpha_b) \rho_g + \rho_b \right] W_{g_m} \left( \frac{\partial h_g}{\partial m} + \frac{\partial h_b}{\partial m} \right) = \Phi_g + \Phi_I + \overline{F}_g \\ + W_{g_m} \frac{\partial p_g}{\partial m} - \frac{\partial \mathcal{E}'_{g_m}}{\partial m} - \frac{\partial \mathcal{E}'_{I_m}}{\partial m} - \frac{\partial \mathcal{E}'_{g_n}}{\partial n} - \frac{\partial \mathcal{E}'_{I_n}}{\partial n} \quad (D-28)$$

Liquid Phase:

1. Droplet - Type 1:

$$\rho_1 W_{p_{m1}} \frac{\partial u_{p1}}{\partial m} = \Phi_{p1} - \Phi_{I1} + \overline{F}_{p1} - \frac{\partial \mathcal{E}'_{p_{m1}}}{\partial m} + \frac{\partial \mathcal{E}'_{I_{m1}}}{\partial m} - \frac{\partial \mathcal{E}'_{p_{n1}}}{\partial n} \\ + \frac{\partial \mathcal{E}'_{I_{n1}}}{\partial n} - M_1 E_{I1} + N_1 \overline{m}_1 E_{I2} + E_{s1} - E_{s2} - E_{\psi_1} \quad (D-29)$$

2. Droplet - Type 2:

$$\rho_2 W_{p_{m2}} \frac{\partial u_{p2}}{\partial m} = \Phi_{p2} - \Phi_{I2} + \overline{F}_{p2} - \frac{\partial \mathcal{E}'_{p_{m2}}}{\partial m} + \frac{\partial \mathcal{E}'_{I_{m2}}}{\partial m} - \frac{\partial \mathcal{E}'_{p_{n2}}}{\partial n} \\ + \frac{\partial \mathcal{E}'_{I_{n2}}}{\partial n} + M_1 E_{I1} - N_1 \overline{m}_1 E_{I2} - E_{s1} + E_{s2} - E_{\psi_2} \quad (D-30)$$



### III. Three-Dimensional Conservation Equations in Cylindrical Coordinates

#### (a) Mass Conservation Equations.

Gas Phase:

$$\begin{aligned} & \frac{1}{r} \frac{\partial}{\partial r} [r \{ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \} W_{gr}] + \frac{1}{r} \frac{\partial}{\partial \theta} [ \{ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \} W_{g\theta} ] \\ & + \frac{\partial}{\partial z} [ \{ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D \} W_{gz} ] + \omega \frac{\partial}{\partial \theta} [ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D ] \\ & = (\dot{m}_{e_1} - \dot{m}_{c_1}) + (\dot{m}_{e_2} - \dot{m}_{c_2}) \end{aligned} \quad (D-31)$$

Liquid Phase

##### 1. Droplet - Type 1:

$$\begin{aligned} & \frac{1}{r} \frac{\partial}{\partial r} (\rho_1 r W_{pr1}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_1 W_{p\theta1}) + \frac{\partial}{\partial z} (\rho_1 W_{pz1}) + \omega \frac{\partial \rho_1}{\partial \theta} \\ & = (\dot{m}_{c_1} - \dot{m}_{e_1}) - M_1 + N_1 \bar{m}_1 \end{aligned} \quad (D-32)$$

##### 2. Droplet - Type 2:

$$\begin{aligned} & \frac{1}{r} \frac{\partial}{\partial r} (\rho_2 r W_{pr2}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho_2 W_{p\theta2}) + \frac{\partial}{\partial z} (\rho_2 W_{pz2}) + \omega \frac{\partial \rho_2}{\partial \theta} \\ & = (\dot{m}_{c_2} - \dot{m}_{e_2}) + M_1 - N_1 \bar{m}_1 \end{aligned} \quad (D-33)$$

#### (b) Momentum Conservation Equations.

Gas Phase:

r-component

$$\begin{aligned} & [ (1 - \sigma_v - \alpha_D) \rho_g + \rho_D ] \left[ W_{gr} \frac{\partial W_{gr}}{\partial r} + \left( \frac{W_{g\theta}}{r} + \omega \right) \frac{\partial W_{gr}}{\partial \theta} \right. \\ & \left. - \left( \frac{W_{g\theta} + r\omega}{r} \right)^2 + W_{gz} \frac{\partial W_{gr}}{\partial z} \right] = - \frac{\partial p_g}{\partial r} + F_{Bg} + F_{Vg} + F_{I_r} \end{aligned} \quad (D-34)$$

z-component

$$\rho_1 \left[ W_{pr1} \frac{\partial W_{pz1}}{\partial r} + \left( \frac{W_{p\theta1}}{r} + \omega \right) \frac{\partial W_{pz1}}{\partial \theta} + W_{pz1} \frac{\partial W_{pz1}}{\partial z} \right]$$

$$= F_{Bpz1} + F_{Vpz1} - F_{Iz1} + \psi_{z1} + N_1 \bar{m}_1 W_{pz2} - M_1 W_{pz1}$$

(D-35)

2. Droplet - Type 2:

r-component

$$\rho_2 \left[ W_{pr2} \frac{\partial W_{pr2}}{\partial r} + \left( \frac{W_{p\theta2}}{r} + \omega \right) \frac{\partial W_{pr2}}{\partial \theta} - \frac{(W_{p\theta2} + r\omega)^2}{r} + W_{pr2} \frac{\partial W_{pr2}}{\partial z} \right]$$

$$= F_{Bpr2} + F_{Vpr2} - F_{Iz2} + \psi_{r2} + M_1 W_{pr1} - N_1 \bar{m}_1 W_{pr2}$$

(D-36)

$\theta$ -component

$$\rho_2 \left[ W_{pr2} \left( \frac{\partial W_{p\theta2}}{\partial r} + \omega \right) + \left( \frac{W_{p\theta2}}{r} + \omega \right) \frac{\partial W_{p\theta2}}{\partial \theta} + W_{pr2} \cdot \right.$$

$$\left. \left( \frac{W_{p\theta2}}{r} + \omega \right) + W_{pz2} \frac{\partial W_{p\theta2}}{\partial z} \right] = F_{Bp\theta2} + F_{Vp\theta2} - F_{I\theta2} + \psi_{\theta2}$$

$$+ M_1 (W_{p\theta1} + r\omega) - N_1 \bar{m}_1 (W_{p\theta2} + r\omega)$$

(D-37)

z-component

$$\rho_2 \left[ W_{pr2} \frac{\partial W_{pz2}}{\partial r} + \left( \frac{W_{p\theta2}}{r} + \omega \right) \frac{\partial W_{pz2}}{\partial \theta} + W_{pz2} \frac{\partial W_{pz2}}{\partial z} \right]$$

$$= F_{Bpz2} + F_{Vpz2} - F_{Iz2} + \psi_{z2} + M_1 W_{pz1} - N_1 \bar{m}_1 W_{pz2}$$

(D-38)

θ-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gr} \left( \frac{\partial W_{g\theta}}{\partial r} + \omega \right) + \left( \frac{W_{g\theta}}{r} + \omega \right) \frac{\partial W_{g\theta}}{\partial \theta} \right. \\ & \left. + W_{gr} \left( \frac{W_{gr}}{r} + \omega \right) + W_{gz} \frac{\partial W_{gz}}{\partial z} \right] = - \frac{1}{r} \frac{\partial p_g}{\partial \theta} + F_{Bg\theta} + F_{Vg\theta} + F_{Ig\theta} \end{aligned} \quad (D-39)$$

z-component

$$\begin{aligned} & \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gr} \frac{\partial W_{gz}}{\partial r} + \left( \frac{W_{g\theta}}{r} + \omega \right) \frac{\partial W_{gz}}{\partial \theta} + W_{gz} \frac{\partial W_{gz}}{\partial z} \right] \\ & = - \frac{\partial p_g}{\partial z} + F_{Bgz} + F_{Vgz} + F_{Igz} \end{aligned} \quad (D-40)$$

Liquid Phase:

1. Droplet - Type 1:

r-component

$$\begin{aligned} & \rho_l \left[ W_{pr1} \frac{\partial W_{pr1}}{\partial r} + \left( \frac{W_{p\theta1}}{r} + \omega \right) \frac{\partial W_{pr1}}{\partial \theta} - \frac{(W_{p\theta1} + r\omega)^2}{r} + W_{pz1} \frac{\partial W_{pr1}}{\partial z} \right] \\ & = F_{Bpr1} + F_{Vpr1} - F_{Ipr1} + \psi_{r1} + N_1 \bar{m}_1 W_{pr2} - M_1 W_{pr1} \end{aligned} \quad (D-41)$$

θ-component

$$\begin{aligned} & \rho_l \left[ W_{pr1} \left( \frac{\partial W_{p\theta1}}{\partial r} + \omega \right) + \left( \frac{W_{p\theta1}}{r} + \omega \right) \frac{\partial W_{p\theta1}}{\partial \theta} + W_{pr1} \left( \frac{W_{p\theta1}}{r} + \omega \right) \right. \\ & \left. + W_{pz1} \frac{\partial W_{p\theta1}}{\partial z} \right] = F_{Bp\theta1} + F_{Vp\theta1} - F_{Ip\theta1} + \psi_{\theta1} + N_1 \bar{m}_1 (W_{p\theta2} + r\omega) \\ & \quad - M_1 (W_{p\theta1} + r\omega) \end{aligned} \quad (D-42)$$

(c) Energy Conservation Equations.

Gas Phase:

$$\begin{aligned}
 & [(1 - \sigma_v - \alpha_b) \rho_g + \rho_b] \left[ W_{gr} \left( \frac{\partial h_g}{\partial r} + \frac{\partial h_b}{\partial r} \right) + \left( \frac{W_{go}}{r} + \omega \right) \cdot \right. \\
 & \left. \left( \frac{\partial h_g}{\partial \theta} + \frac{\partial h_b}{\partial \theta} \right) + W_{gz} \left( \frac{\partial h_g}{\partial z} + \frac{\partial h_b}{\partial z} \right) \right] = \Phi_g + \Phi_I + \bar{P}_g \\
 & + W_{gr} \frac{\partial k_g}{\partial r} + \left( \frac{W_{go}}{r} + \omega \right) \frac{\partial k_g}{\partial \theta} + W_{gz} \frac{\partial k_g}{\partial z} \\
 & - \frac{\partial q'_{gr}}{\partial r} - \frac{\partial q'_{Ir}}{\partial r} - \frac{1}{r} \left[ \frac{\partial q'_{go}}{\partial \theta} + \frac{\partial q'_{Io}}{\partial \theta} \right] - \frac{\partial q'_{gz}}{\partial z} - \frac{\partial q'_{Iz}}{\partial z}
 \end{aligned} \tag{D-43}$$

Liquid Phase:

1. Droplet - Type 1:

$$\begin{aligned}
 \rho_1 \left[ W_{pr1} \frac{\partial u_{p1}}{\partial r} + \left( \frac{W_{po}}{r} + \omega \right) \frac{\partial u_{p1}}{\partial \theta} + W_{pz1} \frac{\partial u_{p1}}{\partial z} \right] &= \Phi_{p1} - \Phi_{I1} + \bar{P}_{p1} \\
 - \frac{\partial q'_{pr1}}{\partial r} - \frac{1}{r} \left( \frac{\partial q'_{po1}}{\partial \theta} - \frac{\partial q'_{Io1}}{\partial \theta} \right) - \frac{\partial q'_{pz1}}{\partial z} + \frac{\partial q'_{Iz1}}{\partial z} \\
 + \frac{\partial q'_{Iz1}}{\partial z} - M_1 E_{I1} + N_1 \bar{m}_1 E_{I2} + E_{s1} - E_{s2} - E_{\psi 1}
 \end{aligned} \tag{D-44}$$

2. Droplet - Type 2:

$$\begin{aligned}
 \rho_2 \left[ W_{pr2} \frac{\partial u_{p2}}{\partial r} + \left( \frac{W_{po2}}{r} + \omega \right) \frac{\partial u_{p2}}{\partial \theta} + W_{pz2} \frac{\partial u_{p2}}{\partial z} \right] &= \Phi_{p2} - \Phi_{I2} \\
 + \bar{P}_{p2} - \frac{\partial q'_{pr2}}{\partial r} - \frac{1}{r} \left( \frac{\partial q'_{po2}}{\partial \theta} - \frac{\partial q'_{Io2}}{\partial \theta} \right) - \frac{\partial q'_{pz2}}{\partial z} \\
 - \frac{\partial q'_{Iz2}}{\partial r} + \frac{\partial q'_{Iz2}}{\partial z} + M_1 E_{I1} - N_1 \bar{m}_1 E_{I2} + E_{s2} \\
 - E_{s1} - E_{\psi 2}
 \end{aligned} \tag{D-45}$$



#### IV. Axisymmetric Conservation Equations in Cylindrical Coordinates

##### (a) Mass Conservation Equations.

Gas Phase:

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left\{ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right\} W_{gr} \right] + \frac{\partial}{\partial z} \left[ \left\{ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right\} W_{gz} \right] \\ = (\dot{m}_{e1} - \dot{m}_{c1}) + (\dot{m}_{e2} - \dot{m}_{c2}) \end{aligned} \quad (D-46)$$

Liquid Phase:

##### 1. Droplet - Type 1:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \rho_l W_{pr1}) + \frac{\partial}{\partial z} (\rho_l W_{pz1}) = (\dot{m}_{c1} - \dot{m}_{e1}) - M_1 + N_1 \bar{m}_1 \quad (D-47)$$

##### 2. Droplet - Type 2:

$$\frac{1}{r} \frac{\partial}{\partial r} (r \rho_2 W_{pr2}) + \frac{\partial}{\partial z} (\rho_2 W_{pz2}) = (\dot{m}_{c2} - \dot{m}_{e2}) + M_1 - N_1 \bar{m}_1 \quad (D-48)$$

##### (b) Momentum conservation Equations.

Gas Phase:

r-component

$$\begin{aligned} \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gr} \frac{\partial W_{gr}}{\partial r} - \frac{(W_{g\theta} + r\omega)^2}{r} + W_{gz} \frac{\partial W_{gz}}{\partial z} \right] \\ = - \frac{\partial P_g}{\partial r} + F_{Bgr} + F_{Vgr} + F_{Irr} \end{aligned} \quad (D-49)$$

$\theta$ -component

$$\begin{aligned} \left[ (1 - \sigma_v - \alpha_b) \rho_g + \rho_b \right] \left[ W_{gr} \left( \frac{\partial W_{g\theta}}{\partial r} + \omega \right) + W_{gz} \left( \frac{\partial W_{g\theta}}{\partial z} + \omega \right) \right. \\ \left. + W_{gz} \frac{\partial W_{gz}}{\partial z} \right] = F_{Bg\theta} + F_{Vg\theta} + F_{I\theta} \end{aligned} \quad (D-50)$$

$$\begin{aligned}
 & \text{z-component} \\
 & \left[ (1 - \sigma_v - \alpha_D) \rho_g + \rho_l \right] \left[ W_{gr} \frac{\partial W_{gz}}{\partial r} + W_{gz} \frac{\partial W_{gz}}{\partial z} \right] \\
 & = - \frac{\partial p}{\partial z} + F_{Bgz} + F_{Vgz} + F_{Igz}
 \end{aligned} \tag{D-51}$$

Liquid Phase:

1. Droplet - Type 1:

$$\begin{aligned}
 & \text{r-component} \\
 & \rho_l \left[ W_{pri} \frac{\partial W_{pri}}{\partial r} - \frac{(W_{p\theta i} + r\omega)^2}{r} + W_{pz i} \frac{\partial W_{pri}}{\partial z} \right] \\
 & = F_{Bpri} + F_{Vpri} - F_{Iri} + \psi_{ri} + N_i \bar{m}_i W_{pr2} - M_i W_{pri}
 \end{aligned} \tag{D-52}$$

$$\begin{aligned}
 & \text{\theta-component} \\
 & \rho_l \left[ W_{pri} \left( \frac{\partial W_{p\theta i}}{\partial r} + \omega \right) + W_{pri} \left( \frac{W_{p\theta i}}{r} + \omega \right) + W_{pz i} \frac{\partial W_{p\theta i}}{\partial z} \right] \\
 & = F_{Bp\theta i} + F_{Vp\theta i} - F_{I\theta i} + \psi_{\theta i} + N_i \bar{m}_i (W_{p\theta 2} + r\omega) \\
 & \quad - M_i (W_{p\theta i} + r\omega)
 \end{aligned} \tag{D-53}$$

$$\begin{aligned}
 & \text{z-component} \\
 & \rho_l \left[ W_{pri} \frac{\partial W_{pz i}}{\partial r} + W_{pz i} \frac{\partial W_{pz i}}{\partial z} \right] = F_{Bpz i} + F_{Vpz i} - F_{Izi} + \psi_{zi} \\
 & \quad + N_i \bar{m}_i W_{pz2} - M_i W_{pz i}
 \end{aligned} \tag{D-54}$$

2. Droplet - Type 2:

$$\begin{aligned} & \text{r-component} \\ \rho_2 \left[ W_{pr2} \frac{\partial W_{pr2}}{\partial r} - \frac{(W_{p\theta2} + r\omega)^2}{r} + W_{pz2} \frac{\partial W_{pr2}}{\partial z} \right] \\ &= F_{Bpr2} + F_{Vpr2} - F_{Ipr2} + \Psi_{r2} + M_1 W_{pr1} - N_1 \bar{m}_1 W_{pr2} \end{aligned} \quad (D-55)$$

$$\begin{aligned} & \text{\theta-component} \\ \rho_2 \left[ W_{pr2} \left( \frac{\partial W_{p\theta2}}{\partial r} + \omega \right) + W_{p\theta2} \left( \frac{W_{p\theta2}}{r} + \omega \right) + W_{pz2} \frac{\partial W_{p\theta2}}{\partial z} \right] \\ &= F_{Bp\theta2} + F_{Vp\theta2} - F_{Ip\theta2} + \Psi_{\theta2} + M_1 (W_{p\theta1} + r\omega) \\ & \quad - N_1 \bar{m}_1 (W_{p\theta2} + r\omega) \end{aligned} \quad (D-56)$$

$$\begin{aligned} & \text{z-component} \\ \rho_2 \left[ W_{pr2} \frac{\partial W_{pz2}}{\partial r} + W_{pz2} \frac{\partial W_{pz2}}{\partial z} \right] &= F_{Bpz2} + F_{Vpz2} - F_{Ipz2} + \Psi_{z2} \\ & \quad + M_1 W_{pz1} - N_1 \bar{m}_1 W_{pz2} \end{aligned} \quad (D-57)$$

(c) Energy Conservation Equations.

Gas Phase:

$$\begin{aligned} & [(1 - \sigma_v - \alpha_b) \rho_g + \rho_o] \left[ W_{gr} \left( \frac{\partial h_g}{\partial r} + \frac{\partial h_p}{\partial r} \right) + W_{gz} \left( \frac{\partial h_g}{\partial z} + \frac{\partial h_p}{\partial z} \right) \right] \\ &= \Phi_g + \Phi_I + \bar{P}_g + W_{gr} \frac{\partial p_g}{\partial r} + W_{gz} \frac{\partial p_g}{\partial z} - \frac{\partial \delta'_{gr}}{\partial r} \\ & \quad - \frac{\partial \delta'_{gr}}{\partial r} - \frac{\partial \delta'_{gz}}{\partial z} - \frac{\partial \delta'_{gz}}{\partial z} \end{aligned} \quad (D-58)$$

Liquid Phase

1. Droplet - Type 1:

$$\begin{aligned} \rho_1 \left[ W_{pr1} \frac{\partial u_{p1}}{\partial r} + W_{pz1} \frac{\partial u_{p1}}{\partial z} \right] &= \Phi_{p1} - \Phi_{I1} + \bar{p}_{p1} - \frac{\partial g'_{pr1}}{\partial r} + \frac{\partial g'_{Ir1}}{\partial r} \\ &\quad - \frac{\partial g'_{pz1}}{\partial z} + \frac{\partial g'_{Iz1}}{\partial z} - M_1 E_{I1} + N_1 \bar{m}_1 E_{I2} + E_{s1} \\ &\quad - E_{s2} - E_{\psi_1} \end{aligned} \quad (D-59)$$

2. Droplet - Type 2:

$$\begin{aligned} \rho_2 \left[ W_{pr2} \frac{\partial u_{p2}}{\partial r} + W_{pz2} \frac{\partial u_{p2}}{\partial z} \right] &= \Phi_{p2} - \Phi_{I2} + \bar{p}_{p2} - \frac{\partial g'_{pr2}}{\partial r} \\ &\quad + \frac{\partial g'_{Ir2}}{\partial r} - \frac{\partial g'_{pz2}}{\partial z} + \frac{\partial g'_{Iz2}}{\partial z} + M_1 E_{I1} - M_1 \bar{m}_1 E_{I2} \\ &\quad + E_{s2} - E_{s1} - E_{\psi_2} \end{aligned} \quad (D-60)$$



## Appendix E

### MODIFICATIONS TO UDO-300 TO ACCOUNT FOR VARIABLE MOLECULAR WEIGHT, SPECIFIC HEATS AND ACOUSTIC VELOCITY

The program UDO-300 utilizes a series of FUNCTION subprograms in determining all thermodynamic and flow properties involving molecular weight, specific heat at constant pressure and acoustic velocity. Each FUNCTION subprogram corresponds to a specific gas-phase property. These subprograms may be separated into three distinct categories, each of which must be handled differently.

1. FUNCTION Subprogram UDG1: used to input certain gas-phase properties to UDO-300. In the original program, UDG1 input the constant value of specific heat at constant pressure, and molecular weight in the form of a gas constant. Since both these properties are variable for two-phase flow considerations, UDG1 will now input the compressor inlet values of specific heat at constant pressure and molecular weight. These will serve as reference values. Also, certain properties pertaining to the particulate phase will be input by this subprogram for use in the calculation of the gas-phase acoustic velocity.
2. FUNCTION Subprograms UDG3-UDG5 and UDG7- UDG9: output certain gas properties using input of enthalpy. Each of these subprograms determines a result from an equation of the form:

$$x = \phi(c_{p_g}) f(mw_g) \delta(H) g(?)$$

where,  $x$  = desired flow property

$\phi(c_{p_g})$  = dependence of  $x$  on  $c_{p_g}$

$f(mw_g)$  = dependence of  $x$  on  $mw_g$

$\delta(H)$  = dependence of  $x$  on enthalpy,  $H$

$g(?)$  = some arbitrary function unique to each subprogram providing equality (may involve functions of additional flow properties).

Using the perfect gas assumption, that the gas-phase specific heat and recalling and molecular weight are given as functions of the gas-phase mixture temperature only, then

$$c_{p_g} = H/T_g = f(H/c_{p_g}) = f_1(H) \quad (E-1)$$

and

$$mw_g = mw_g(T_g) = mw_g(c_{p_g}) = f_2(H). \quad (E-2)$$

An iterative technique is needed to determine  $c_{p_g}$  and  $mw_g$  due to the nature of equations (E-1) and (E-2), given only the enthalpy. An addition to these subprograms that is needed to account for variable specific heat and molecular weight, therefore consists in the iterative technique.

In the case of subprogram UDG9,  $g(?)$  involves the acoustic velocity. An extra section is added to this subprogram to account for droplet affects on the gas-phase speed of sound.

3. FUNCTION Subprograms UDG2 and UDG6: output enthalpy using input of certain gas properties. In general, an iteration scheme is employed. Enthalpy is determined using the reference value of  $c_{p_g}$  and  $mw_g$ , input by UDG1. Using the equations (E-1) and (E-2) of the iteration technique involved in the previous FUNCTION subprogram discussion, the values of  $c_{p_g}$  and  $mw_g$  corresponding to the just calculated enthalpy can be compared with the

the reference values. A value of enthalpy is then output when the iteration scheme converges on the gas-phase specific heat and molecular weight.

#### ANALYSIS:

##### (a) Variable molecular weight and specific heats

- Subroutine ITER: This subroutine is called by FUNCTION subprograms UDG2 - UDG5 and UDG7 - UDG9. A value of enthalpy is input to the subroutine and corresponding gas-phase specific heat at constant pressure and molecular weight are determined by an iterative approach. These values of  $c_{p_g}$  and  $mw_g$  are returned to the calling FUNCTION subprogram by the labeled common "GAS". It is assumed that the functions of  $c_{p_g}$  and  $mw_g$  given by equations (E-1 ) and ( E-2 ) are well behaved, and convergence always occurs.
- Subroutine SET: The purpose of SET is to retain the reference values of  $c_{p_g}$  and  $mw_g$ , input by UDG1, that correspond to the compressor inlet conditions. These reference values are used in the first iteration performed by ITER. The final iteration fixes different values for  $c_{p_g}$  and  $mw_g$  corresponding to the input enthalpy and transmits these back to the calling FUNCTION subprogram by the labelled common "GAS".
- Subroutine RETURN: This subroutine accepts the reference values of gas-phase specific heat and molecular weight stored by SET and re-establishes these values in the labeled common "GAS".

Thus, whenever ITER is called, the following procedure occurs:

- (i) Reference  $c_{p_g}$  and  $mw_g$  values are retained by SET
- (ii) ITER determines  $c_{p_g}$  and  $mw_g$  corresponding to input enthalpy using reference  $c_{p_g}$  and  $mw_g$  in first iteration
- (iii) ITER returns values of  $c_{p_g}$  and  $mw_g$  found in (ii) to calling FUNCTION subprogram by way of "GAS"
- (iv) Subroutine RETURN resets the values of  $c_{p_g}$  and  $mw_g$  in common "GAS" to reference values stored by SET.

(b) Acoustic Velocity

Equation (D-1) and the liquid-phase properties  $\rho_p$  and  $c_p$  (introduced to the subprogram by the labeled common "LIQ", and now input to the program UDO-300 by the subprogram UDG1) are added to the subprogram UDG9.

Modifications to UDO-300

```

FUNCTION UDG1 (LNLT)
common/GAS/G,EJ,R,CP,ROJCP
common/LIQ/RHOP,AD
LOG 1 = 1
LOG 2 = 2
READ (LOG 1, 100) CP,R,G,EJ
READ (LOG 1, 101) RHOP,AP
100 FORMAT (4F12.0)
101 FORMAT (2F12.0)
  If (CP,EQ,0.0) CP = 0.24
  If (R,EQ,0.0) R = 53.32
  If (G,EQ,0.0) G = 32.174
  If (EJ,EQ,0.0) EJ = 778.16
WRITE (LOG 2,110) CP,R,G,EJ,RHOP,AP
110 FORMAT (2x,/,10x,37H INLET SPECIFIC HEAT AT CONST
  1 PRESSURE,2x,1H =,F8.5,/,10x,18H INLET GAS CONSTANT,21x,
  2 1H =,F8.4,/,10x,22H GRAVITATIONAL CONSTANT, 17x,1H =,F8.4,
  3 /,10x,17H JOULES EQUIVALENT,22x,1H =, F8.3,/,10x,20H LIQUID
  4 PHASE DENSITY,19x,1H =,F8.4,/,10x,24H LIQUID ACOUSTIC VELOCITY,
  5 15x,1H =,F8.2)
  LNCT = LNCT + 5
  ROJCP = R/(EJ x CP)
  RETURN
END

```



```

FUNCTION UDG2(S,P)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
TOLRR = .001
I = 1
1 H = CP*EXP(S/CP + ROJCP*ALOGCP)
CALL ITER(H)
If(I.NE.1)GO TO 2
HO = H
I = I + 1
GO TO 1
2 If((ABS(H - HO)/H).LE.TOLRR)GO TO 3
GO TO 1
3 UDG2 = H
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

FUNCTION UDG3(P,H)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
UDG3 = CP*ALOG(H/CP) - R/EJ*ALOG(P)
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

FUNCTION UDG4(H,S)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
UDG4 = EXP(ALOG(H/CP)/ROJCP - EJ/R*S)
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

FUNCTION UDG5(H,S)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
UDG5 = UDG4(H,S)/(R*H)*CP
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

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ANALYSIS OF WATER INGESTION EFFECTS IN AXIAL FLOW COMPRESSORS.(U)

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FUNCTION UDG6(P,T)
CP = f(T)
UDG6 = CP*T
RETURN
END

```

\*Install

```

FUNCTION UDG7(H,S)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
UDG7 = H/CP
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

FUNCTION UDG8(H,S)
common/GAS/G,EJ,R,CP,ROJCP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
UDG8 = 1.0/(1.0 - ROJCP)
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

FUNCTION UDG9(H,S,V2)
common/GAS/G,EJ,R,CP,ROJCP
common/LIQ/RHOP,AP
CALL SET(RO,ROJCO,CPO)
CALL ITER(H)
GAMMA = 1.0/(1.0 - ROJCP)
AG = CP/(GAMMA*G*R*H)
PG = EXP(ALOG(H/CP)/ROJCP - EJ/R*S)
TG = H/CP
RHOG = PG/(R*TG)
SIGV = f(H,S) [+  $\sigma_v = \sigma_v(H,S)$ ]
AM1 = (1.0 - SIGV)*RHOG + SIGV*RHOP
AM2 = (1.0 - SIGV)/(RHOG*AG**2.) + SIGV/(RHOP*AP**2.)
AM = (AM1*AM2)**-.5
UDG9 = V2/AM
CALL RETURN(RO,ROJCO,CPO)
RETURN
END

```

```

SUBROUTINE SET(RO,ROJCO,CPO)
common/GAS/G,EJ,R,CP,ROJCP
RO = R
ROJCO = ROJCP
CPO = CP
RETURN
END

```

```

SUBROUTINE RETURN(RO,ROJCO,CPO)
common/GAS/G,EJ,R,CP,ROJCP
R = RO
ROJCP = ROJCO
CP = CPO
RETURN
END

```

```

SUBROUTINE ITER(H)
common/GAS/G,EJ,R,CP,ROJCP
TOLRR = .001
1 I = 1
2 TG1 = H/CP
3 CP1 = CP1(TG1)(Install)
  If(ABS((CP - CP1)/CP1).LE.TOLRR)GO TO 5
  If(I.GT.1)GO TO 4
  DELT2 = CP1 - CP
  CP = CP/1.1
  TG2 = TG1
  I = 2
  GO TO 2
4 DELT1 = CP1 - CP
  DELTT = TG1 - TG2
  TG2 = TG1
  TG1 = TG1 - DELTT/(DELT1 - DELT2)*DELT1
  DELT2 = DELT1
  CP = H/TG1
  GO TO 3
5 R1 = R1(TG1)(Install)
  TG = H/CP1
  R = R(TG)(Install)
  If(ABS((R1 - R)/R).LE.TOLRR)GO TO 6
  TOLRR = TOLRR/2.0
  GO TO 1
6 ROJCP = R/(EJ*CP1)
RETURN
END

```



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